Math 204 HOMEWORK #1 Name____________________

1/8/18 [/10]

Show all your work for full credit on a separate sheet(s) of paper. Unsupported answers = reduced points.

1. Find the values for α and β such that the system of equations represented by the augmented matrix $\begin{pmatrix} 2 & \beta & 3 \\ \alpha & -3 & 5 \end{pmatrix}$ is:

a) consistent

b) dependent

c) inconsistent

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2. Find the value(s) of k such that the system of equations represented by the augmented matrix is consistent, and explain geometrically what the system represents.

$$\begin{pmatrix} 3 & 4 & k \\ 1 & 1 & 2 \\ 2 & k & 9 \end{pmatrix}$$

[/2]

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3. Row reduce the matrix to reduced echelon form, and circle the pivot positions in the final matrix and the original matrix. Does the matrix represent a consistent, inconsistent, or dependent system?

$$\begin{pmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 8 & 4 & 2 \end{pmatrix}$$

[/1]

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4. **TRUE / FALSE** (justify your reasons)

a. If every column of an augmented matrix contains a pivot, then the corresponding system is consistent.

b. The reduced echelon form of a matrix is unique.

c. Whenever a system has free variables, there is a unique solution.

d. The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.

e. A general solution of a system is an explicit description of all solutions of the system.

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5. Find the general solution to the system of equations. Show all your row-equivalent matrices.

$$x_1 - 4x_2 + 4x_3 + 2x_4 = 1$$
$$2x_1 - 8x_2 + 7x_3 + 6x_4 = -4$$

[/1]

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6. Write the system of equations in #5 above as a vector equation.

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7. Determine if the vector **b** is a linear combination of vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. (You can use **rref**)

$$\mathbf{a}_1 = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}, \quad \mathbf{a}_2 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \quad \mathbf{a}_3 = \begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 4 \\ -18 \\ 10 \end{pmatrix}$$

[/1]

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8. Find the value of h so that the vector **w** is in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$, then write **w** as a linear combination of $\mathbf{v}_1$, and $\mathbf{v}_2$.

$$\mathbf{w} = \begin{pmatrix} 3 \\ h \\ -4 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$$

[/2]
