Math 146 **9.1 - Tests for Two Proportions**

EXAMPLE 1 **Bednets to Reduce Malaria** In a randomized controlled trial in Kenya, insecticide- treated bednets were tested as a way to reduce malaria. Among 343 infants using bednets, 15 developed malaria. Among 294 infants not using bednets, 27 developed malaria (based on data from "Sustainability of Reductions in Malaria Transmission and Infant Mortality in Western Kenya with Use of Insecticide-Treated Bednets," by Lindblade et al., Journal of the American Medical Association, Vol. 291, No. 21). We want to use a 0.01 significance level to test the claim that the incidence of malaria is lower for infants using bednets.

1. Bednets
$n_1 = 343$
$x_1 = 15$
$\hat{p}_1 = \frac{15}{343}$
$\hat{p}_1 = 0.044$

2. No bednets
$n_2 = 294$
$x_2 = 27$
$\hat{p}_2 = \frac{27}{294}$
$\hat{p}_2 = 0.092$

$H_0: p_1 = p_2$
$H_1: p_1 < p_2$ claim
$\alpha = 0.01$

Hypothesis Test for Two Proportions

	Sample 1	Sample 2
x=	15	27
n=	343	294
Sample Proportions=	0.0437	0.0918
Pooled Proportion=	0.0659	
Type:	Ha: p1 - p2 < 0	
z-statistic=	-2.4389	
P-value=	0.0074	

NORMAL FLOAT AUTO REAL RADIAN MP

2-PropZTest
x1:15
n1:343
x2:27
n2:294
p1:≠p2 <p2 >p2
Color: BLUE
Draw

NORMAL FLOAT AUTO REAL RADIAN MP

2-PropZTest
p1<p2
z=-2.438917655
p=0.0073656638
p̂1=0.0437317784
p̂2=0.0918367347
p̂=0.0659340659
n1=343
n2=294

$\bar{p} = \frac{15+27}{343+294}$

$\bar{p} \approx 0.066$

Since $p < \alpha$ Reject H_0.

There is sufficient statistical evidence to support the claim that bednets reduce the incidence of malaria in infants.

Create 98% confidence interval ($\alpha = 0.02$)

$\hat{p}_1 = 0.044$ $\hat{p}_2 = 0.092$

$n_1 = 343$ $n_2 = 294$

$z_{\alpha/2} = 2.326$

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$= 2.326\sqrt{\frac{(0.044)(0.956)}{343} + \frac{(0.092)(0.908)}{294}}$$

$E = 0.047$

$\hat{p}_1 - \hat{p}_2 = 0.044 - 0.092$

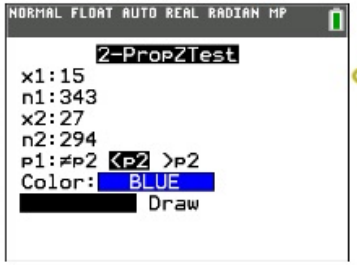
$= -0.048$

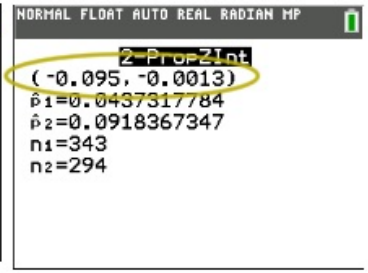
CI: $\hat{p}_1 - \hat{p}_2 - E < p_1 - p_2 < \hat{p}_1 - \hat{p}_2 + E$

$-0.048 - 0.047 < p_1 - p_2 < -0.048 + 0.047$

$$-0.095 < p_1 - p_2 < -0.001$$

T184 2-PropZInt





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Confidence Interval	
Confidence Level	0.98
-0.0950	-0.0013

Warning: conclusion may not be the same as when using the p-value.

EXAMPLE 2 **Police Gunfire** In a study of police gunfire reports during a recent year, it was found that among 540 shots fired by New York City police, 182 hit their targets; and among 283 shots fired by Los Angeles police, 77 hit their targets (based on data from the New York Times). We want to use a 0.05 significance level to test the claim that New York City police and Los Angeles police have the same proportion of hits.

a. Test the claim using a hypothesis test.

b. Test the claim by constructing an appropriate confidence interval. (Warning: this does not always agree with the conclusion using the P-value method.)

① NY ② LA

$n_1 = 540$ $n_2 = 283$

$x_1 = 182$ $x_2 = 77$

$H_0: p_1 = p_2$ claim

$H_1: p_1 \neq p_2$

$\alpha = 0.05$

$\hat{p}_1 = \frac{182}{540} \approx 0.337$ $\hat{p}_2 = \frac{77}{283} \approx 0.272$

$P > \alpha$ Fail to reject H_0

There is not enough evidence to refute the claim that the two police departments have the same proportion of hits.

$-0.0005 < p_1 - p_2 < 0.1304$

Hypothesis Test for Two Proportions

	Sample 1	Sample 2
x=	182	77
n=	540	283
Sample Proportions=	0.3370	0.2721
Pooled Proportion=	0.3147	
Type:	Ha: p1 - p2 ≠ 0	
z-statistic=	1.9059	
P-value=	0.0567	

Confidence Interval	
Confidence Level	0.95
-0.0005	0.1304

Warning: conclusion may not be the same as when using the p-value.