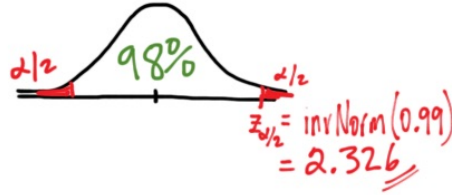


7.2 - Estimating a Population Mean

EXAMPLE 1 Suppose the distribution of male heights has $\sigma = 2.8$ inches. A sample of 50 males had a sample mean of $\bar{x} = 69.2$ inches. Calculate a confidence interval to estimate the population mean height, μ , using $\alpha = 0.02$.

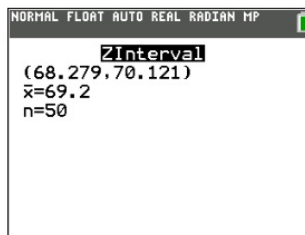
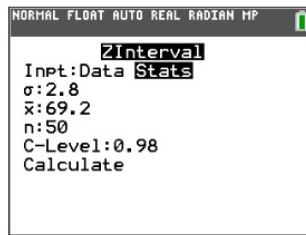
From CLT $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \Rightarrow E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$$E = 2.326 \cdot \frac{2.8}{\sqrt{50}} = 0.921$$

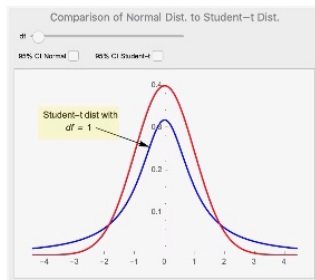


CI : $\bar{x} \pm E = 69.2 \pm 0.921$

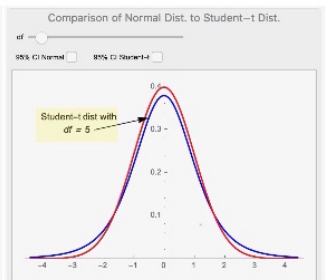
$\Rightarrow 68.3 < \mu < 70.1$



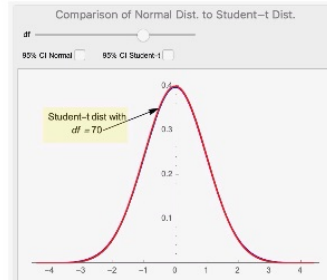
WHEN σ IS NOT KNOWN WE USE THE t-distribution



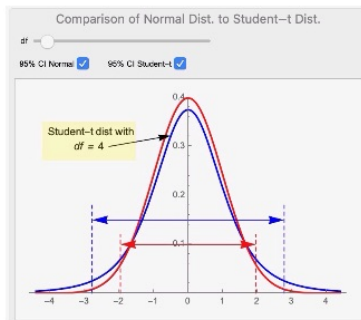
DF=1 \Rightarrow n=2



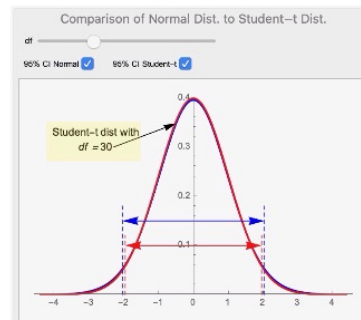
DF=5 \Rightarrow n=6



DF=70 n=71



DF=4 n=5



DF=30 \Rightarrow n=31

EXAMPLE 2 Find the critical number for $\alpha = 0.05$ and $n = 20$ using Table A-3 and $\text{invT}(\alpha/2, df)$.

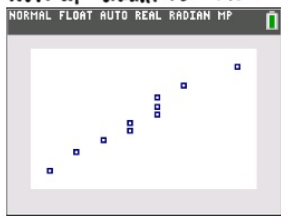
From table A-3 with $DF=19$, an area 0.05 in TWO tails
 $t_{\alpha/2} = 2.093$ TI-84 $\text{invT}(0.975, 19) = 2.09302$

EXAMPLE 3 The speed limit in front of a school zone is 25 mph, and the speed of ten cars were recorded:

23 24 22 20 27 24 25 21 23 24

Assuming the speeds are normally distributed (is this a valid assumption?) find a 95% confidence interval for the mean speed.

Normal-Quantile Plot



1-Var Stats
 $\bar{x} = 23.3$
 $\Sigma x = 233$
 $\Sigma x^2 = 5465$
 $Sx = 2.002775851$
 $\sigma x = 1.9$
 $n = 10$
 $\text{minX} = 20$
 $\downarrow Q1 = 22$

$\bar{x} = 23.3$
 $s = 2.003$
 $n = 10$
 $df = 9$
 $t_{\alpha/2} = 2.262$
 $CI: \bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
 $= 23.3 \pm 2.262 \cdot \frac{2.003}{\sqrt{10}}$
 $= 23.3 \pm 1.433$
 $21.87 < \mu < 24.73$

Estimating Means Using Data

Data:	Sample Mean =	23.3000
1 23	Sample St. Dev. =	2.0028
2 24	Sample Size =	10
3 22	Confidence Level =	0.95
4 20	alpha/2 =	0.025
5 27	Critical Value $t_{\alpha/2}$ =	2.2622
6 24	MOE =	1.4327
7 25	95%-Confidence Interval	
8 21	21.8673	24.7327
9 23		
10 24		
11		

EXAMPLE 4 An earlier study on the GPA of college students found the standard deviation to be 0.513 and a mean of 3.179. How many students would need to be surveyed to find a 97% confidence interval for the mean with an error of $E = 0.05$. Calculate the confidence interval.

$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ since n is not known we can't find $t_{\alpha/2}$. So, approximate using:
 $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \Rightarrow n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$
 For a 97% CI $z_{\alpha/2} = 2.1701$
 $\therefore n = \left(\frac{(2.1701)(0.513)}{0.05}\right)^2$
 $n = 495.7$
 $n = 496$

$CI = 3.179 \pm 0.05$
 $3.129 < \mu < 3.229$

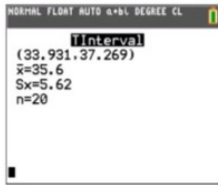
Sample Size for Means

Standard Deviation =	0.513
Confidence Level =	0.97
alpha/2 =	0.015
Margin of Error =	0.05
Critical Value =	2.1701
Sample Size	496

Estimating Means Using Statistics

Sample Mean =	3.179
Sample St. Dev. =	0.513
Sample Size =	496
Confidence Level =	0.97
alpha/2 =	0.015
Critical Value $t_{\alpha/2}$ =	2.1764
MOE =	0.0501
97%-Confidence Interval	
3.1289	3.2291

EXAMPLE 5 I calculated a confidence interval on my calculator, but forgot what α -level I used. Can you find it?



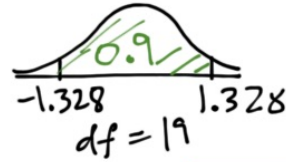
$$MOE = \frac{37.269 - 33.931}{2}$$

$$= 1.669$$

$$E = \frac{t_{\alpha/2} s}{\sqrt{n}}$$

$$1.669 = \frac{t_{\alpha/2} (5.62)}{\sqrt{20}}$$

$$1.328 = t_{\alpha/2}$$



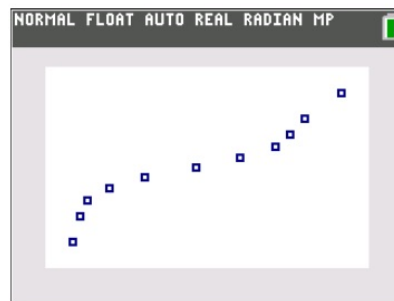
90% CI $\Rightarrow \alpha = 0.1$

EXAMPLE 6 Find a 95% confidence interval for the population mean using the following sample data set, and give a statement about the interval.

13 26 31 5 35 4 3 33 8 40 20

Estimating Means Using Data

Data:		Sample Mean=	19.8182
1	13	Sample St. Dev.=	13.8334
2	26	Sample Size=	11
3	31	Confidence Level=	0.95
4	5	alpha/2=	0.025
5	35	Critical Value $t_{\alpha/2}$ =	2.2281
6	4	MOE=	9.2934
7	3	95%-Confidence Interval	
8	33	10.5248	29.1116
9	8		
10	40		
11	20		



Not normal and $n < 30$ so the confidence interval is unreliable!