

7.1 - Estimating a Population Proportion

From our survey  $n = 30$  students,  $x = 17$  females, so

$$\hat{p} = \frac{17}{30}$$

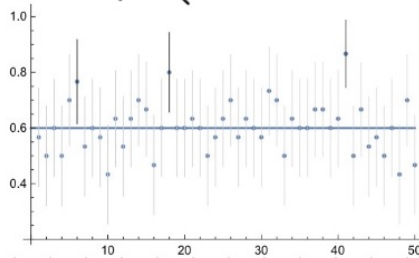
$$\hat{p} = 0.57$$

$\pm 0.05$

90% confident it's between 52% and 62%

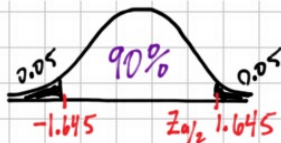
95% confident  $47\% < p < 67\%$

99% confident  $37\% < p < 77\%$



**EXAMPLE 2** A simple random sample of 300 registered voters (not a call in, self selected, or observational sample, etc.) at a political polling station resulted in 168 individuals that voted for a democrat. Find a confidence interval at the  $\alpha = 0.10$  confidence level for the true proportion of democratic voters.

$$\alpha/2 = 0.05$$



$$\hat{p} = \frac{168}{300}$$

$$\hat{p} = 0.56$$

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 1.645 \cdot \sqrt{\frac{(0.56)(0.44)}{300}}$$

$$\approx 0.047$$

so CI:  $\hat{p} \pm E = 0.56 \pm 0.047$

$$\Rightarrow 0.513 < p < 0.607$$

Using EasyCalc

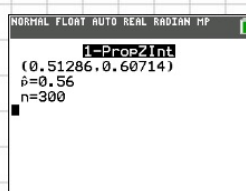
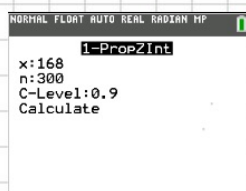
Estimating Proportions

Sample Size: n=	300
x=	168
p-hat=	0.5600
Confidence Level=	0.9
alpha/2=	0.05

Critical Value=	1.6449
MOE=	0.0471

90%-Confidence Interval	
0.5129	0.6071

ON THE TI-84



**EXAMPLE 3** A television station claimed in a recent poll only 34% Seattleites supported a new gasoline tax. The poll had a margin of error of  $\pm 3\%$  at a 95% confidence level. Find the sample size of the poll assuming they had prior knowledge that  $\hat{p} \approx 0.3$ . Round **up** if  $n$  is not an integer.

Using the formula:  $n = \frac{(z_{\alpha/2})^2 \hat{p} \hat{q}}{E^2}$        $z_{\alpha/2} = \text{invNorm}(0.975) = 1.960$   
 $\hat{q} = 1 - 0.3 = 0.7$   
 $n = \frac{1.96^2 (0.3)(0.7)}{0.03^2}$   
 $\approx 896.37$   
 **$n = 897$**

EasyCalc:

Sample Size for Proportions	
Estimate for p-hat=	0.3 (or leave blank)
Confidence Level=	0.95
alpha/2=	0.025
Margin of Error=	0.03
Critical Value=	1.9600
<b>Sample Size</b>	
897	

**EXAMPLE 4** A sample of 500 people showed that 82% watched the Super Bowl.

a) Find a 95% confidence interval for the proportion of people that watched the Super Bowl.

We need  $x$ :  $x = 500(0.82)$   
 $x = 410$

b) Suppose we had no idea how many people watched the Super Bowl. How many people would we need to randomly sample to have a 98% confidence that our calculated confidence interval contained  $p$  within  $\pm 3\%$ ? (Typical polls have a  $\pm 3\%$  error using a 95% confidence interval. Find this sample size.)

a)

Estimating Proportions	
Sample Size: n=	500
x=	410
p-hat=	0.8200
Confidence Level=	0.95
alpha/2=	0.025
Critical Value=	1.9600
MOE=	0.0337
<b>95%-Confidence Interval</b>	
0.7863      0.8537	

**$0.786 < p < 0.854$**

b) use  $\hat{p} \hat{q} = 0.25$

Sample Size for Proportions	
Estimate for p-hat=	(or leave blank)
Confidence Level=	0.98
alpha/2=	0.01
Margin of Error=	0.03
Critical Value=	2.3263
<b>Sample Size</b>	
1504	