6.6 - Normal Approximation to a Binomial

Example 1 For the binomial distribution $B(30, 0.4)$ Find:

a) $P(x = 10)$ using the binomial distribution, and

b) approximate $P(x = 10)$ using the normal distribution.

$n = 30$
$p = 0.4$

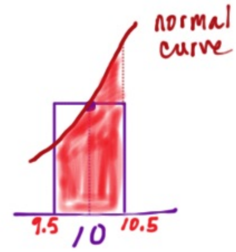
① $np = 30(0.4) = 12$ ✓
$nq = 30(0.6) = 18$ ✓

$\mu = 12$
$\sigma = \sqrt{npq} = \sqrt{30(0.4)(0.6)} \approx 2.68328$

a) $P(x=10) = \text{binompdf}(30, 0.4, 10)$
$\boxed{P = 0.11519}$

b) $P(x=10) \approx \text{normalcdf}(9.5, 10.5, 12, 2.68328)$
$\boxed{P \approx 0.11233}$

Fairly close.



Example 2 For the the binomial distribution $B(50, 0.23)$ calculate $P(15 \le x \le 30)$ using the binomial distribution, and approximating it with a normal distribution.

$np = 50(0.23) = 11.5 \geq 5$ ✓

$P(15 \le x \le 30) = \text{binomcdf}(50, 0.23, 30) - \text{binomcdf}(50, 0.23, 14)$
$\boxed{P = 0.1565}$

$\mu = 50(0.23) = 11.5$ $\sigma = \sqrt{50(.23)(.77)} = 2.9757$

$P(15 \le x \le 30) = \text{normalcdf}(14.5, 30.5, 11.5, 2.9757)$
$\boxed{P = 0.1567}$

Example 3 The probability of a baby being born a male is 0.512. In one thousand births, approximate the probability of having 520 or more males. Compare with the binomial calculation.

$n = 1000$
$x = 520$
$p = 0.512$

$\mu = 1000(0.512) = 512$
$\sigma = \sqrt{1000(0.512)(0.488)} = 15.807$

$P(x \geq 520) = \text{normalcdf}(519.5, 1E99, 512, 15.807)$
$\boxed{P = 0.318}$

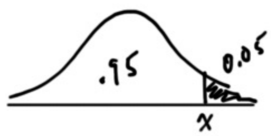
Compare to binomial:

$P(x \geq 520) = 1 - \text{binomcdf}(1000, 0.512, 519)$
$\boxed{P = 0.318}$

Example 4 For the binomial distribution $B(200, 0.05)$ approximate P_{95}.

$\mu = 200(0.05)$
$= 10$ ✓
≥ 5

$\sigma = \sqrt{200(0.05)(.95)}$
$= 3.08221$



$x \approx \text{invNorm}(0.95, 10, 3.08221)$
≈ 15.07

Since the binomial is discrete we should round this up to 16.

So $P_{95} = 16$

Check