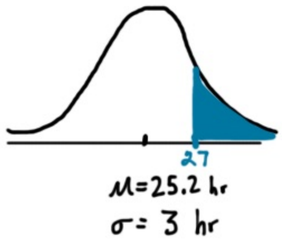
6.4 - The Central Limit Theorem

EXAMPLE 1 A.C. Nielsen reported that children between the ages of 2 and 5 watch an average of 25.2 hours of television per week. Assume the distribution is normally distributed and the standard deviation is 3 hours.

a) Find the probability that if a single child is randomly selected, the mean number of hours they watch television is greater than 27 hours. (Draw a normal curve.)

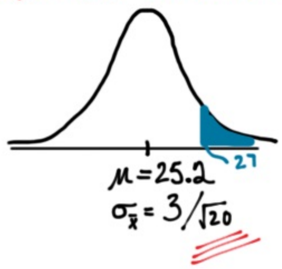


$P(x > 27) = \text{normalcdf}(27, 1E99, 25.2, 3)$

$\boxed{P = 0.274}$ quite likely.

b) If 20 children between the ages of 2 and 5 are randomly selected, find the probability that the sample mean of all 20 children is greater than 27 hours of television watched.

$n < 30$ but we were told the distribution is normal, so we can use CLT.

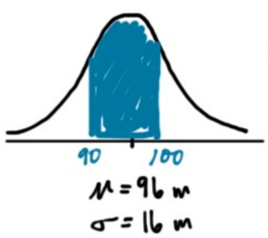


$P(\bar{x} > 27) = \text{normalcdf}(27, 1E99, 25.2, 3/\sqrt{20})$

$\boxed{P = 0.0036}$ not very likely!

EXAMPLE 2 The average age of a vehicle registered in the United States is 8 years, or 96 months. Assume the standard deviation is 16 months.

a) Assuming the ages are normally distributed, find the probability that a randomly selected vehicle will have an age between 90 and 100 months.

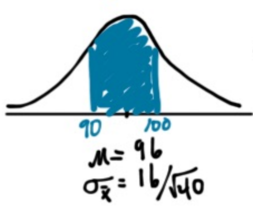


$P(90 < x < 100) = \text{normalcdf}(90, 100, 96, 16)$

$\boxed{P = 0.245}$

b) If a random sample of 40 vehicles is selected, find the probability that the mean of their age is between 90 and 100 months.

normality is not mentioned, but $n > 30$, use CLT.



$P(90 < \bar{x} < 100) = \text{normalcdf}(90, 100, 96, 16/\sqrt{40})$

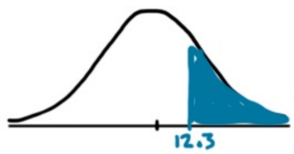
$\boxed{P = 0.934}$

c) Find the probability that a sample of 10 cars has a mean age of more than 9 years.

Normality is unknown and $n < 30$. Can't do any calculations. :(

EXAMPLE 3 If, under a certain assumption, the probability of an observed event is extremely small, say $p < 0.05$, we conclude that the assumption is probably not correct. A machine fills 12-oz water bottles in a bottling plant, and has a standard deviation of 0.2 oz. If the machine is determined to be over-filling bottles, the machine line needs to be shut down and recalibrated. Let's assume the machine is filling bottles with a mean volume of 12 oz.

a) Assuming a normal distribution, a single bottle of water is tested and found to have a mean of 12.3 oz. Should the bottling line be shut down and recalibrated?



$\mu = 12$ oz

$\sigma = 0.2$ oz

$P(x > 12.3) = \text{normalcdf}(12.3, 1E99, 12, 0.2)$

$\boxed{P = 0.0668}$

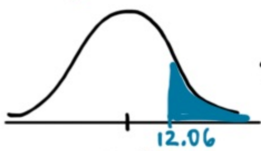
This is not less than 0.05 and is expected due to randomness.

b) A sample of 20 bottles is found to have a mean of 11.5 oz. Is recalibration necessary?

Since $n < 30$ and we don't know if the dist. is normal, we can't use CLT.

c) A sample of 50 bottles is found to have a mean of 12.06 oz. Is recalibration necessary.

$n > 30$, use CLT.



$\mu = 12$

$\sigma_{\bar{x}} = \frac{0.2}{\sqrt{50}}$

$P(\bar{x} > 12.06) = \text{normalcdf}(12.06, 1E99, 12, 0.2/\sqrt{50})$

$\boxed{P = 0.017}$

Since $P < 0.05$ we conclude our assumption that the machine is filling with a mean of 12oz is wrong. Recalibrate!!