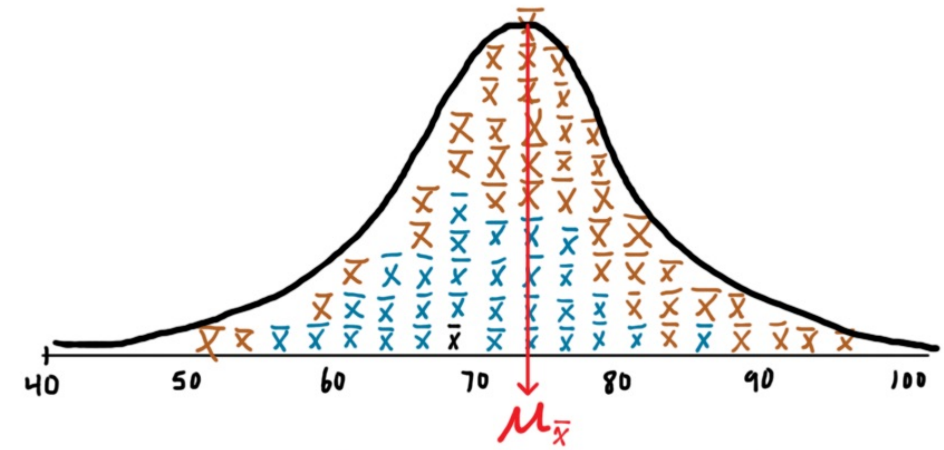
6.3 - SAMPLING DISTRIBUTIONS

HEART RATE : $n = 30$ $\bar{x} = 69.7$ But what is μ?



QUESTION: Does $\mu_{\bar{x}} = \mu$?

ALSO: WHAT IS $\sigma_{\bar{x}}$?

Math 146 6.3 - Sampling Distributions and Estimators

The sampling distribution of a statistic is the distribution of all values of the statistic when all possible samples of the same size *n* are taken from the same population. For example, the sampling distribution of the sample mean is the distribution of sample means, with all samples having the same size *n* taken from the same population.

Suppose a *population* has a distribution with mean μ and standard deviation σ. If we take all simple random samples (SRS) of size *n* and make a probability histogram of the sample means $\bar{x}$, will the mean of the sample means be the same as the mean of the population? That is, will $\mu_{\bar{x}} = \mu$, and hence $\mu_{\bar{x}}$ is a good estimator for μ?

EXAMPLE 1 Suppose a class of 3 students (a very small population) is given a 10 point worksheet and the results of the worksheets were 2, 5, and 11 points.

(a) Find the mean, population variance, standard deviation, median, Mode, range, and proportion of values greater than 9. These are all *parameters* since they describe the *population*.

POPULATION = $\{2, 5, 11\}$ Median : $\{2, 5, 11\}$

$\mu = \frac{2+5+11}{3} = \underline{\underline{6}}$ $\sigma^2 = \frac{4^2 + 1^2 + 5^2}{3} = 14$ Range $= 11 - 2 = 9$ $P = \frac{1}{3}$

$\boxed{\mu = 6}$ $\boxed{\sigma^2 = 14}$ $\boxed{\sigma = \sqrt{14}}$ $\boxed{\text{MED} = 5}$ $\boxed{\text{MODE} = \text{None}}$ $\boxed{\text{RNG} = 9}$ $\boxed{P = \frac{1}{3}}$

≈ 3.7417

POPULATION PARAMETERS

(b) List all possible simple random samples of size two from the three worksheet values **with replacement** (why?) and calculate the sample mean, variance, standard deviation, median, mode, range, and proportion of the sample with values greater than 9. (There should be 9 different pairs.) 2, 5, 11

Samples	Mean ($\bar{x}$)	Var. (s^2)	St. Dev. (s)	Median	Mode	Range	Prop ($\hat{p}$)
2, 2	2	0	0	2	2	0	0
2, 5	3.5	4.5	2.1213	3.5	—	3	0
2, 11	6.5	40.5	6.3640	6.5	—	9	0.5
5, 2	3.5	4.5	2.1213	3.5	—	3	0
5, 5	5	0	0	5	5	0	0
5, 11	8	18	4.2426	8	—	6	0.5
11, 2	6.5	40.5	6.3640	6.5	—	9	0.5
11, 5	8	18	4.2426	8	—	6	0.5
11, 11	11	0	0	11	11	0	1
means →	6	14	~~2.8284~~	~~6~~	~~6~~	~~4~~	$\frac{1}{3}$
	$\boxed{\mu = 6}$	$\boxed{\sigma^2 = 14}$	~~$\boxed{\sigma = \sqrt{14}}$~~ ≈ 3.7417	~~$\boxed{MED = 5}$~~	~~$\boxed{MODE = None}$~~	~~$\boxed{RNG = 9}$~~	$\boxed{p = \frac{1}{3}}$

(c) Find the *mean* of each of the statistics above. Which means look like they *target* the population parameter?

An *estimator* is a *statistic* used to infer, or estimate, the value of a population parameter. An *unbiased* estimator targets the value of the population parameter, whereas a biased estimator does not.

(d) What can be said about the *estimators* in part (c)?

The unbiased estimators are:

1) sample <u>means</u>
2) sample <u>variance</u>
3) sample <u>proportions</u>

Meaning: $\boxed{\mu_{\bar{x}} \to \mu \quad \mu_{s^2} \to \sigma^2 \quad \mu_{\hat{p}} \to p}$

(e) Find the standard deviation of the sample means, $\sigma_{\bar{x}}$, in (b) and compare to $\frac{\sigma}{\sqrt{n}}$.

The standard deviation of all 9 sample means is:

$\boxed{\sigma_{\bar{x}} = 2.64575}$ From the original population of 3 values we had $\sigma = \sqrt{14}$

so, $\frac{\sigma}{\sqrt{n}} = \frac{\sqrt{14}}{\sqrt{2}} = \sqrt{7} = 2.64575$!! $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$