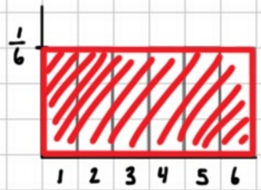
MATH 146 6.1 Continuous Probability Distributions - The Standard Normal Distribution

UNIFORM DISTRIBUTIONS

Create a probability distribution for the outcomes of rolling a single die.

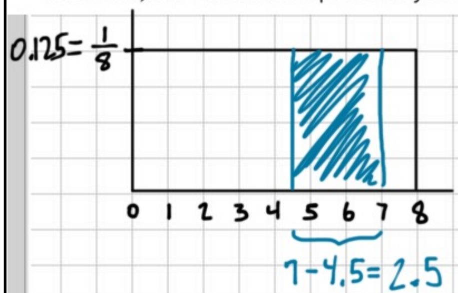
x	$P(x)$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6



$\text{Area} = 6 \cdot \frac{1}{6}$

$= 1$ or 100%

EXAMPLE 1 Suppose a bus leaves Skagit Valley College every eight minutes. This means a persons wait time will always be between 0 and 8 minutes, and is uniformly distributed. Draw the *probability density function* for wait times, and calculate the probability of having to wait between 4.5 and 7 minutes.



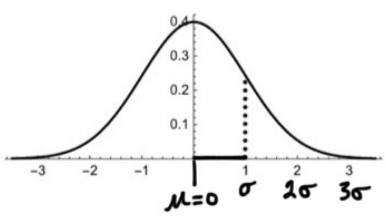
$P(4.5 \le x \le 7) = 2.5(0.125)$

$= 0.3125$

About 31% chance of waiting between 4.5 to 7 minutes for the bus.

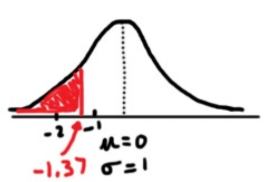
THE STANDARD NORMAL DISTRIBUTION

1) The density function for the standard normal function is bell-shaped,
2) has parameters $\mu = 0$ and $\sigma = 1$; $N(0, 1)$
3) the total area under the curve from $-\infty < z < \infty$ equals 1.



EXAMPLE 2 Use Table A-2 (back of the text) to calculate the following areas. Draw a normal curve and the indicated area.

a) Less than $z = -1.37$



NEGATIVE z Scores

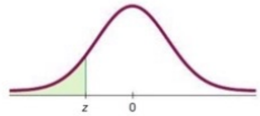
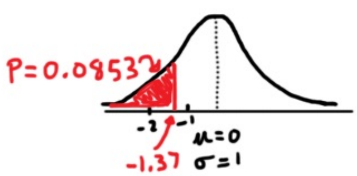


TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

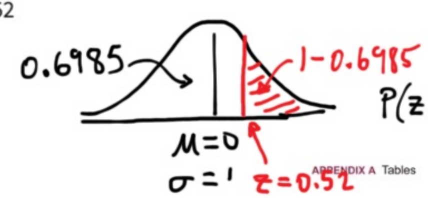
EXAMPLE 2 Use Table A-2 (back of the text) to calculate the following areas. Draw a normal curve and the indicated area.

a) Less than $z = -1.37$

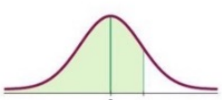


$P(z \le -1.37) = 0.0853$

b) Greater than $z = 0.52$



$P(z \ge 0.52) = 1 - 0.6985$
$= 0.3015$

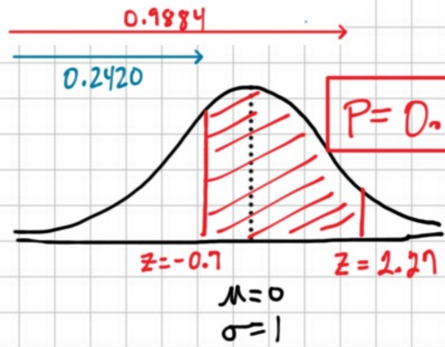


POSITIVE z Scores

TABLE A-2 *(continued)* Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549

c) Between $z = -0.70$ and $z = 2.27$



P = 0.7464

FROM TABLE A-2

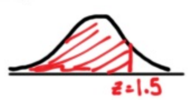
$P(z \le 2.27) = 0.9884$

$P(z \le -0.7) = 0.2420$

$P(-0.7 \le z \le 2.27) = 0.9884 - 0.2420$
$= 0.7464$

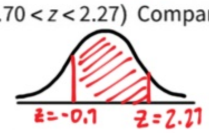
EXAMPLE 3 A normal distribution has mean $\mu = 0$ and standard deviation $\sigma = 1$. Calculate the following probabilities and use a normal curve to indicate the value.

a) $P(z < 1.5)$



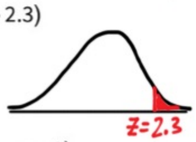
normalcdf(-1E99, 1.5) = **0.9332**

b) $P(-0.70 < z < 2.27)$ Compare with part (c) above.



normalcdf(-0.7, 2.27) = **0.7464**

c) $P(z > 2.3)$



normalcdf(2.3, 1E99) = **0.0107**

d) $P(-2 < z < 2)$

normalcdf(-2, 2) = **0.9545**

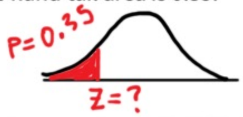
Recall the Emperical Rule: 68-95-99.7

Finding z-scores From Areas

On the TI-84, **invNorm (*p*)** calculates the *z*-score that will give a left-hand tail area of *p*.

EXAMPLE 4 Find the following *z*-scores for the given areas (probabilities):

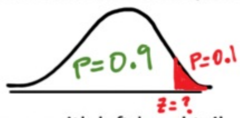
a) Left-hand tail area is 0.35.



invNorm(0.35) = −0.385

$z = -0.385$

b) Right-hand tail area is 0.10 (Note: this is the 90th percentile, or P_{90}).



invNorm(0.9) = 1.282

$z = 1.282$

c) The *z*-scores with left-hand tail area 0.025, and right-hand-tail area 0.025.

z_1 = invNorm(0.025)

$z_1 = -1.960$

$\Rightarrow z_2 = 1.960$

or z_2 = invNorm(0.975)

$z_2 = 1.960$