Math 146 5.2 - Binomial Distributions

Example 1: In a litter of 5 Labrador puppies there is a 35% chance that one is golden and a 65% chance that it is black. Identify: S, F, n, p, and q, and find P(x=2) where x is the number of golden labs.

Since $x$ = # of golden Labs then

S = getting a golden F = getting a black lab , $n=5$ $p=0.35$ $q=0.65$

$$P(x) = \frac{n!}{(n-x)!\,x!} p^x q^{n-x} \qquad n=5 \quad x=2 \quad p=0.35 \quad q=0.65$$

$$P(2) = \frac{5!}{(5-2)!\,2!}(0.35)^2(0.65)^3$$

$$= \frac{5\cdot4\cancel{\cdot3\cdot2\cdot1}}{(\cancel{3\cdot2\cdot1})(2\cdot1)}(0.35)^2(0.65)^3$$

$$= \frac{20}{2}(0.35)^2(0.65)^3$$

$$= 10(0.35)^2(0.65)^3$$

$$= 0.3364 \approx \boxed{33.6\%}$$

USING THE TI-84

1. Press 2nd dist
2. scroll down to binompdf

binompdf(n, p, x)

binompdf(5, 0.35, 2) ENTER

$\approx 0.33642$

Example 2 In packaging eggs, it is found that an egg has a probability of 10% of being damaged. The company packages eggs in cartons of 12 (1 dozen).
Identify: S, F, n, p, and q

S = A damaged egg F = a good egg. $n=12$ $p=0.1$ $q=0.9$

a. $P(x=3)$ means the probability of getting 3 damaged eggs in 1 dozen.

S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$P(x=3)$ = binompdf(12, 0.1, 3)

= 0.085

b. $P(x \le 3)$ this means the probability of at most 3 damaged eggs.

S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$P(x \le 3)$ = binomcdf(12, 0.1, 3)

= 0.974

c. $P(x \ge 7)$ This is the probability of at least 7 damaged eggs.

S = {0, 1, 2, 3, 4, 5, 6, $\boxed{7, 8, 9, 10, 11, 12}$}

1 − P( )

$P(x \ge 7)$ = 1 − binomcdf(12, 0.1, 6) = 0.00005 VERY UNLIKELY !!

d. $P(2 \leq x \leq 6)$ The probability of getting between 2 and 6 (inclusive) bad eggs.

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

cdf

− cdf

leaves [2 3 4 5 6]

$P(2 \leq x \leq 6) = \text{binomcdf}(12, 0.1, 6) - \text{binomcdf}(12, 0.1, 1)$

$= 0.341$

$\approx 34.1\%$ of having between 2 and 6 damaged eggs! :(

Example 3

Corona Virus Example:

$n = 50$, $p = 0.02$ $\Rightarrow$ $B(50, 0.02)$ distribution

a. $P(x=3) = \text{binompdf}(50, 0.02, 3)$

$= 0.139$

$\approx 13.9\%$ chance of exactly 3 infected in a group of 50.

b. $P(x \geq 6) = 1 - \text{binomcdf}(50, 0.02, 5)$

$= 0.3839$

$\approx 38.4\%$ chance of at least 6 infected in a group of 50.

EXAMPLE 4

Find the mean and standard deviation of a $B(5, 0.35)$ distribution.

$\mu = np$ $\qquad$ $\mu = 5(0.35) = 1.75$

$\sigma^2 = npq$ $\qquad$ $\sigma = \sqrt{5(0.35)(0.65)}$

$\sigma = \sqrt{npq}$ $\qquad$ $\approx 1.07$

Binompdf(5,0.35)

| X | P(x) |
|---|---|
| 0 | 0.116 |
| 1 | 0.312 |
| 2 | 0.336 |
| 3 | 0.181 |
| 4 | 0.049 |
| 5 | 0.005 |

Unusual values

$\mu \pm 2\sigma = 1.75 \pm 2(1.07)$
$\approx (-0.39, 3.89)$

Having 0 goldens would not be unusual, but having 4 or 5 goldens would be unusual!