

Show all your work for full credit. Unsupported answers = reduced points. Clearly identify your answers; use additional paper if necessary.

1. For small, slowly falling objects, the assumption made in the text that the drag force is proportional to the velocity is a good one. For larger, more rapidly falling objects, it is more accurate to assume that the drag force is proportional to the square of the velocity.

- (a) Write a differential equation for the velocity of a falling object of mass m if the drag force is proportional to the square of the velocity.
- (b) Determine the limiting velocity after a long time.
- (c) If $m = 10$ kg, find the drag coefficient so that the limiting velocity is 49 m/s.

[/3]

2. Consider the differential equation $dy/dt = a y - b$.

- (a) Find the equilibrium solution y_e
- (b) Let $Y(t) = y - y_e$; thus $Y(t)$ is the deviation from the equilibrium solution. Find the differential equation satisfied by $Y(t)$.

[/3]

3. Determine which of the following equations is a solution to the differential equation:

$$t^2 y'' + 5 t y' + 4 y = 0$$

$$y_1(t) = t^{-2} \ln(t)$$

$$y_2(t) = t^{-1} \ln(t)$$

$$y_3(t) = t^{-2}$$

[/2]

4. Find the value(s) of r for which the function $y = e^{rt}$ is a solution to the differential equation:
 $2y'' + y' - 10y = 0$

[/2]

5. Determine the values of r for which $y = t^r$ is a solution to the equation $t^2 y'' - 4t y' + 4y = 0$.

[/3]

6. Show that the multivariable equation $u = \left(\frac{\pi}{t}\right)^{1/2} e^{-x^2/(4\alpha^2 t)}$, for $t > 0$, is a solution to the partial differential equation

$$\alpha^2 u_{xx} = u_t.$$

[/2]
