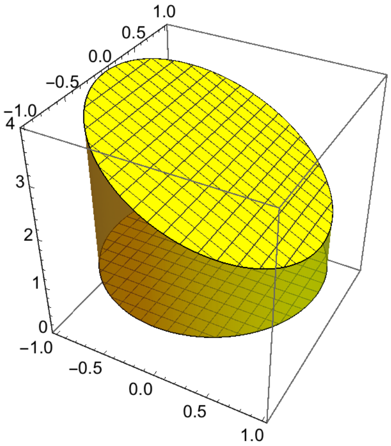
16.9 The Divergence Theorem (Gauss's Divergence Theorem)

Gauss's Divergence Theorem: Let E be a simple solid region and let S be the boundary surface of E with positive (outward) orientation. Let $\boldsymbol{F}$ be a vector field with continuous partial derivatives in an open region that contains E. Then

$$\iint_S \boldsymbol{F}\cdot d\boldsymbol{S} = \iint_S \boldsymbol{F}\cdot\boldsymbol{n}\,dS = \iiint_E \nabla\cdot\boldsymbol{F}\,dV = \iiint_E \operatorname{div}\boldsymbol{F}\,dV$$

Example 1 Verify the Divergence Theorem for $\boldsymbol{F}(x, y, z) = 4x\,\boldsymbol{i} - 2y^2\,\boldsymbol{j} + z^2\,\boldsymbol{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$, and $z = 3$.

Example 2 Verify the Divergence Theorem for the surface of the solid bounded by $x^2 + y^2 = 1$, $z = 3 - x$, and $z = 1$ and $\mathbf{F}(x, y, z) = y\,\mathbf{i} + x\,\mathbf{j} - z\,\mathbf{k}$.



Vector Plot

Example 3 Evaluate $\iint\limits_S \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F}(x, y, z) = 4xz\,\mathbf{i} - y^2\,\mathbf{j} + yz\,\mathbf{k}$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$.