

15.3 Volume of Solids with Non-Rectangular Bases

Suppose a solid has a non-rectangular base bounded by two functions $y_1 = g_1(x)$ and $y_2 = g_2(x)$ where $g_1(x) < g_2(x)$ for all $x \in [a, b]$, e.g., the base may have the shape below in Figure 1, and a height $z = f(x, y)$. A possible solid is shown in Figure 2.

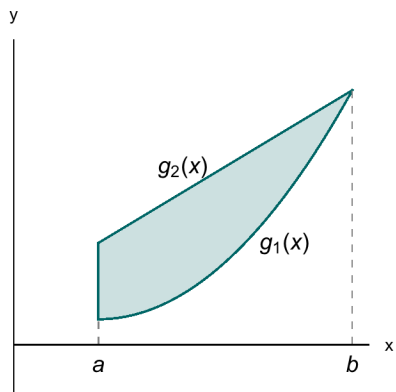


Figure 1

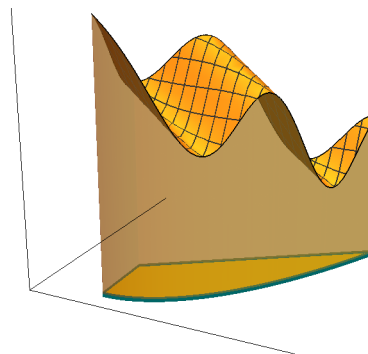
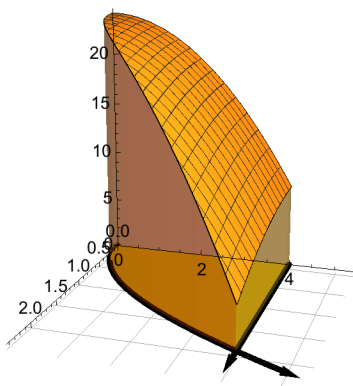


Figure 2

Using the tried and true concept of “slice, approximate, and integrate”, we can slice the base region parallel to the y -axis from g_1 to g_2 . These slices run in the x direction from a to b , and the height at each point is $f(x, y)$. This leads to the expression:

$$\text{Volume} = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \, dy \, dx$$

Example 1 Find the volume of the solid that has a top surface given by $f(x, y) = 24 - x^2 - y^2$, and base in the first quadrant bounded by $y = x^2$ and $y = 4$.



Example 2 Graph the base region of the solid given by $\int_0^2 \int_0^{x^3} f(x, y) dy dx$, and change the order of integration.

Example 3 Find the value of $\int_0^4 \int_{x/2}^2 e^{y^2} dy dx$.

Example 4 Evaluate: $\int_0^{\pi/2} \int_y^{\pi/2} \frac{\sin(x)}{x} dx dy$