14.5 Chain Rule and Implicit Differentiation

Example 1 Let $f(x, y) = z$ and $x = g(t)$ and $y = h(t)$. Find $\frac{df}{dt}$.

Example 2 Let $f(x, y) = z$ and $x = g(s, t)$ and $y = h(s, t)$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

Example 3 Let $f(x, y) = x\sin(xy)$, $x = \sqrt{st}$, and $y = t^2 - s$. Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$ when $t = 1$ and $s = 4$.

Example 4 Given the ideal gas law: $PV = nRT$, find the rate the volume is changing if the pressure is 50 Pa and is decreasing 2 Pa/min, and the temperature is 300 K and is decreasing 5 K/min. Assume $n = 10$ moles of gas, and $R = 8.3144\ J/(K \cdot \text{mol})$.

Another Look at Implicit Differentiation

Suppose a function in terms of x and y, *i.e.* $y = f(x)$, is written implicitly as $F(x, y) = 0$.

Example 5 Find $\frac{dy}{dx}$ in terms of F_x and F_y.

Example 6 Assume the function $z = f(x, y)$ is written $F(x, y, z) = 0$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Example 7 Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^2 y + x z y = 4 y z^2$.

Example 8 Suppose that the partial derivatives of $f(x, y, z)$ are continuous and that

$$f_x(1, 1, 1) = 4, \quad f_y(1, 1, 1) = -5, \quad \text{and } f_z(1, 1, 1) = 3$$

Let $g(t) = f\left(t^4, t^5, t^6\right)$. Find $g'(1)$.