Math 254 Calculus IV Notes

14.1 Functions of Several Variables

Domain

A function of two variables maps an ordered pair (x, y) in the domain to a value z in the range: $z = f(x, y)$.

Example 1 Find and sketch the domain for the function: $f(x, y) = \frac{\sqrt{x - y^2 + 4}}{x - 1}$

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Example 2 Find and sketch the domain for $f(x, y) = \ln(2x + y)$

The Graph of a Function of Several Variables

The graph of a function $f(x, y) = z$ is the set of all points (x, y, z) in $\mathbb{R}^3$, such that (x, y) is in the domain of f.

Example 3 Make a sketch of the function $f(x, y) = \sqrt{x^2 + y^2}$

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Example 4 Make a sketch of the function $f(x, y) = x^2 + y$.

Example 5 Make a graph of the function $f(x, y) = \cos(x^2 + y^2)$ and restrict the domain to a circle of radius 2 centered at (1, 2).

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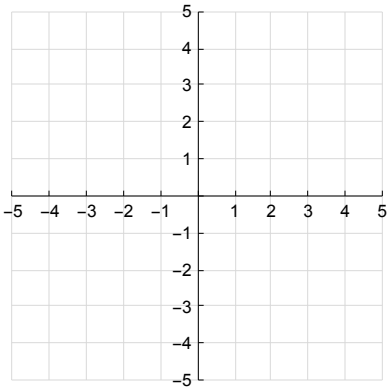
Level Curves (or Contour Curves)

The level curves of a function f in two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f.)

Example 6 Graph the level curves for the function: $f(x, y) = x^2 - y$ for $k = \{-4, -2, 0, 2, 4\}$.



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Example 7 For the function $f(x, y) = (x^2 + 3y^2 - x)e^{-x^2-y^2}$ plot the function and contours for $k \in [-0.2, 1.2]$.

Example 8 Describe the level contours in $\mathbb{R}^3$ for $f(x, y, z) = x^2 + y^2 + z^2$.