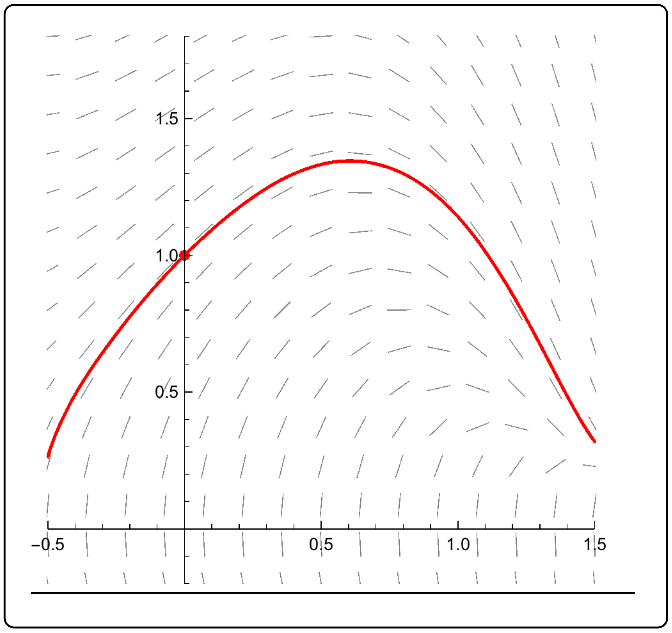
8.1-2 Numerical Solutions: Euler's Method

Consider the IVP $y' = -2t^2 + 1/y$, $y(0) = 1$, and the need to find $y(0.5)$. Unfortunately an algebraic solution doesn't exist.

```
DSolve[{y'[t] == -2 t^2 + 1 / y[t], y[0] == 1}, y[t], t] // Simplify
```

```
DSolve[{2 t^2 + y'[t] == 1/y[t], y[0] == 1}, y[t], t]
```

The direction field and solution curve appears to give $y(0.5) \approx 1.3$, and numerically the solution is $y(0.5) \approx 1.33656$.



Euler's Method is a simple numerical method that uses the tangent line at the point in the direction field to approximate a new point. Then, a new approximation is made using the previous point, and the process is repeated until the desired estimate is reached.

Example 1 Calculate the first several points for the differential equation above: $y' = -2t^2 + 1/y$, $y(0) = 1$, using $h = 0.1$.

Example 2 Use Euler's method to approximate $y(2)$ for the initial value problem $y' = 2t + y$ where $y(1) = 4$ using a step size $h = 0.2$, and compare it to the exact value $y(2) = 8e - 6$.

Pseudo Code

```
Step 1. define f(t, y)
Step 2. input initial values t0 and y0
Step 3. input step size h and number of steps n
Step 4. output t0 and y0
Step 5. for j from 1 to n do
Step 6.  k1 = f(t, y)
         y = y + h * k1
         t = t + h
Step 7. output t and y
Step 8. end
```

TI-84 Euler's Method Program

A very simple program for Euler's method for the TI-84 is:

```
PROGRAM: EULER
    Prompt T,Y,H,N
    For(I,1,N)
    Y+Y1 H → Y
    T+H → T
    Disp {T,Y}
    Pause   (This step is optional)
    End
```

Enter $y'(t, y)$ into **Y1** and then run **EULER**. Enter the four required values and keep pressing enter.

Example 3 Use the Euler program (or equivalent) to solve the IVP: $y' + 2ty = 5t$, $y(1) = 3$, for $y(2)$ using $h = 0.1$.

The Improved Euler Method

An improvement on Euler's method is to use the average of the values at the two endpoints of the interval $[t_n, t_{n+1}]$. (This takes into consideration, somewhat, the concavity of the integral curve). This results in a few additional computations in step 6 from above:

```
Step 6.  k1 = f(t, y)
         k2 = f(t + h, y + h * k1)
         y = y + (h/2) * (k1 + k2)
         t = t + h
```

Example 4 Write an Improved Euler's Method for the TI-84 and numerically solve problem 2.