

7.8 Systems of Equations with Repeated Eigenvalues

Example 1 Find the eigenvalues and eigenvectors for the matrix $A = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$ and propose a solution, $\mathbf{x} = c_1 \boldsymbol{\xi}_1 + c_2 \boldsymbol{\xi}_2$, to the equation $\mathbf{x}' = A \mathbf{x}$.

Example 2 Determine if $\boldsymbol{\xi}_2 = t \mathbf{v} e^{\lambda t}$ is a possible second solution to the equation $\mathbf{x}' = A \mathbf{x}$.

Example 3 Show that $\boldsymbol{\xi}_2 = t \mathbf{v} e^{\lambda t} + \mathbf{p} e^{\lambda t}$ is a solution to the equation $\mathbf{x}' = A \mathbf{x}$ for an appropriate vector \mathbf{p} , and find a method to find \mathbf{p} .

Example 4 Solve the initial value problem $\mathbf{x}' = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix} \mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

Example 5 Solve the initial value problem $\mathbf{x}' = \begin{pmatrix} 7 & 1 \\ -4 & 3 \end{pmatrix} \mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$