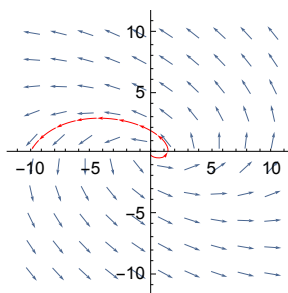## 7.6 Systems of Equations with Complex Eigenvalues
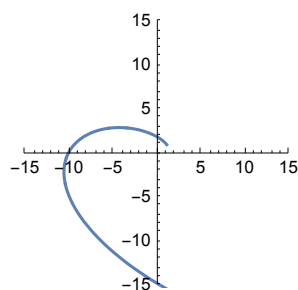
The direction field and solution curve for the system of equations $\quad \mathbf{x'} = \begin{pmatrix} 1 & -5 \\ 2 & 3 \end{pmatrix} \mathbf{x}; \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \;$ is

The solution using *Mathematica* is:

```
sol = DSolve[{x1'[t] == x1[t] - 5 x2[t], x2'[t] == 2 x1[t] + 3 x2[t], x1[0] == 1, x2[0] == 1},
    {x1[t], x2[t]}, t] // Flatten
```

$\left\{ x1[t] \rightarrow e^{2t} (Cos[3t] - 2 Sin[3t]), \; x2[t] \rightarrow e^{2t} (Cos[3t] + Sin[3t]) \right\}$

Example 1 Find the eigenvalues and eigenvectors for the example above.

Example 1 cont.   Solve the initial value problem above.

Example 2   Solve the initial value problem $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix}\mathbf{x};\quad \mathbf{x}(0) = \begin{pmatrix} 5 \\ 5 \end{pmatrix}.$

Example 3   Plot the solution and a phase portrait for $\mathbf{x}' = \begin{pmatrix} 4 & -10 \\ 2 & -4 \end{pmatrix}\mathbf{x};\quad \mathbf{x}(0) = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$