

7.2/3 Matrices; Eigenvalues and Eigenvectors

Example 1 Write the differential equation $\begin{cases} x' = x + 2y \\ y' = 2x + y \end{cases}$ as a matrix equation.

Example 2 Show that the vector $\mathbf{x} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} e^{2t}$ is a solution to the equation $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$.

Example 3 Solve the system of equations or show there is no solution:

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 1 \\ x_1 - x_2 + 2x_3 &= 1 \end{aligned}$$

Example 4 If an $n \times n$ matrix A has an inverse A^{-1} , then $AA^{-1} = A^{-1}A = I_n$. Show that the 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has the inverse $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Example 5 Solve the system of equations $\begin{cases} 3x - 5y = 4 \\ 2x + 4y = 11 \end{cases}$ algebraically, and using the inverse of the matrix A when the system is written in the form of $A\mathbf{x} = \mathbf{b}$.

Eigenvalues and Eigenvectors

Consider the equation $A\mathbf{x} = \lambda\mathbf{x}$ for a square matrix A , vector \mathbf{x} , and a scalar λ . The vector \mathbf{x} is called an eigenvector of matrix A , and the associated eigenvalue is λ .

Example 6 Show that the matrix $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, has eigenvalues $\lambda_1 = 5$, and $\lambda_2 = 1$ with associated eigenvectors $v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, and $v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Example 7 Find the eigenvalues and eigenvectors for the matrix $A = \begin{bmatrix} 5 & -2 \\ 1 & 2 \end{bmatrix}$

Example 8 Find the eigenvalues and eigenvectors for the matrix $A = \begin{bmatrix} 3 & 2 \\ 12 & -2 \end{bmatrix}$.