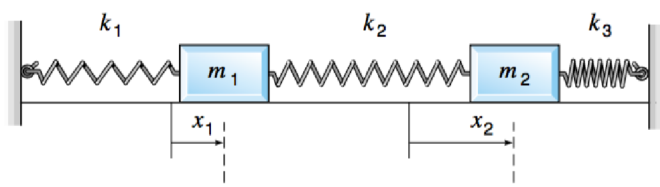
7.1 Systems of Differential Equations

A system of differential equations arises when two or more quantities are related to each other. Recall that the rate of change of a population is proportional to the size of the population, we get the differential equation $p'(t) = k\,p(t)$. If we have two interacting populations, say wolves (x) and rabbits (y) a system of equations relating their individual growth rates is:

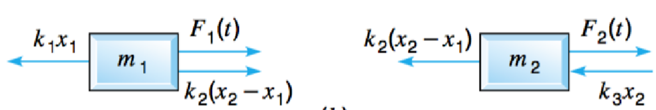
$$\begin{cases} x' = (-a + b\,y)\,x \\ y' = \quad (c - d\,x)\,y \end{cases}$$

Example 1 Write the second order differential equation as a first order linear system: $y'' + 4\,t\,y' + t^2\,y = 0$

Example 2 Use the free-body diagram to set-up the equations for the position of each mass of the two-mass three-spring system.



(*a*)



(*b*)

Example 3 Solve the system of differential equations by rewriting the system as a second order equation:

$$\begin{cases} x' = 2x + y \\ y' = x + 2y \end{cases} \quad x(0) = 10 \text{ and } y(0) = 20.$$

Example 4 Suppose two tanks, X and Y, each with an acid solution, are coupled together. Tank X is a 100 L tank with 300 g of acid initially, and is being filled with fresh water at a rate of 2 L/min. A pipe flows solution from tank X to tank Y at a rate of 5 L/min, and a second pipe circulates solution from tank Y to tank X at the 3 L/min. Tank Y is a 50 L tank with an initial quantity of 500 g of acid and is also being drained at 2 L/min. Set up a system of equations to determine the quantity of acid in each tank at any time t. Graph both solutions and find the maximum concentration of acid in each tank.