6.5 Impulse Functions

An impulse functions is an instantaneous forcing function. Consider the function $f(t) = \frac{1}{2\epsilon}(u_{-\epsilon}(t) - u_{\epsilon}(t))$ on the interval $(-\infty, \infty)$. As $\epsilon \to 0$, the area will always be 1, and therefore the total impulse $I(\epsilon) = \int_{-\infty}^{\infty} f(t)\,dt = 1$, and $\lim_{\epsilon \to 0} I(\epsilon) = 1$ even though $\lim_{\epsilon \to 0} f(t) = 0$ for all $t \neq 0$.

The equations above can be used to define an idealized **unit impulse function** δ that has a magnitude of of 1 at $t = 0$ but is zero everywhere else.

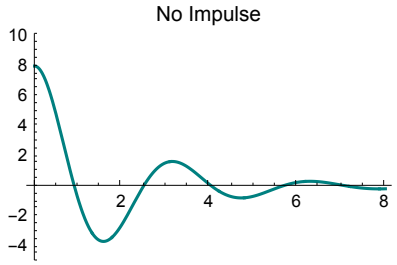
$$\delta(t) = 0 \text{ when } t \neq 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

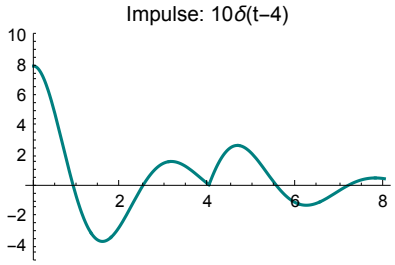
Also, $\delta(t - c)$ is a unit impulse at $t = c$, and $\delta(t - c) = 0$ whenever $t \neq c$.

The Laplace of the Unit Impulse Function (the Dirac delta function)

$$\mathcal{L}\{\delta(t - c)\} = e^{-sc}$$

Example 1 Consider the equation: $y'' + y' + \frac{17}{4}y = 0$; $y(0) = 8$, $y'(0) = 0$ whose solution is given. Find the solution is if the system is given an impulse of 10 at time $t = 4$.





Example 2 Consider the vibration given by: $4y'' + y = h(t)$, $y(0) = 10$, $y'(0) = 0$.

a) Solve the IVP if $h(t) = 20\,\delta(t - 3\pi)$.

b) Can you find an impulse at $t = 3\pi$ that will perfectly stop the vibration?