

6.4 Solving Equations with Discontinuous Forcing Functions

Example 1 Solve the equation: $y'' + 3y' + 2y = h(t)$; $y(0) = 8$, $y'(0) = 0$, $h(t) = \begin{cases} 4t & 0 \leq t < 1 \\ 4 & 1 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$

Example 2 Solve the equation: $2y'' + 2y' + 5y = h(t)$; $y(0) = 10$, $y'(0) = 0$, and $h(t) = \begin{cases} 10 & 5 \leq t < 10 \\ 0 & \text{elsewhere} \end{cases}$

Example 3 Find the Laplace transform output, $Y(s)$, for $y'' + y' + y = f(t)$, $y(0) = 0$, $y'(0) = 0$;

$$f_T(t) = \begin{cases} 1 & 0 \leq t < 1 \\ -1 & 1 \leq t < 2 \\ 0 & \text{elsewhere} \end{cases} \quad . \text{ (Do not solve)}$$

Example 4 Use *Mathematica* to solve $2y'' + y' + 30y = f(t)$; $y(0) = 20$, $y'(t) = 0$ where f is a constant function of 150 between $t = 5$ and $t = 8$. Graph the unforced function, the forcing function f , and the forced function.