

## 3.6 Non-homogeneous Equations; Method of Undetermined Coefficients

**Example 1** Find a general solution to the non-homogeneous equation:  $y'' + 2y' - 8y = 3e^{-2t}$ . Begin by find the solution to the homogeneous equation, and then make an “educated guess” for another possible solution.

### Method of Undetermined Coefficients

To solve the non-homogeneous equation  $ay'' + by' + cy = g(t)$ ,

1. Find the solution to the homogeneous equation,  $y_h$ .
2. Find a particular solution to the non-homogeneous,  $y_p$ , using  $g(t)$  to find undetermined coefficients (see below).
3. The general solution to the non-homogeneous equation is given by  $y = y_h + y_p$ .

The particular solution to the non-homogeneous equation depends on the form of  $g(t)$ :

If $g(t)$ has the form	the particular solution, $y_p$ , will have the form
1: $P_n(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0$	$t^s (A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0)$
2: $P_n(t) e^{\alpha t}$	$t^s (A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0) e^{\alpha t}$
3: $\cos(\beta t)$ and/or $\sin(\beta t)$	$t^s (B_1 \cos(\beta t) + B_2 \sin(\beta t))$
4: $P_n(t) \begin{pmatrix} \cos(\beta t) \\ \text{or} \\ \sin(\beta t) \end{pmatrix} e^{\alpha t}$	$t^s [(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0) e^{\alpha t} \cos(\beta t) + (B_n t^n + B_{n-1} t^{n-1} + \cdots + B_0) e^{\alpha t} \sin(\beta t)]$

(Choose  $s$ , so that there are no common terms in  $y_p$  and  $y_h$ .  $s$  will be 0, 1, or 2)

**Example 2** Find the general solution to the equation  $y'' - 5y' + 6y = 2t^2 + 3$

**Example 3** Find the general solution to the equation  $y'' + 3y' - 4y = 10 \sin(2t)$

**Example 4** Find the general solution to the equation  $y'' - 6y' + 9y = 8e^{3t}$

**Example 5** Find the solution to the equation  $y'' + 4y = 4 \cos(2t)$ ,  $y(0) = 10$  and  $y'(0) = 1$