3.5 Non-homogeneous Equations; Method of Undetermined Coefficients

Example 1 Find a general solution to the non-homogeneous equation: $y'' + 2y' - 8y = 3e^{-2t}$. Begin by find the solution to the homogeneous equation, and then make an "educated guess" for another possible solution.

Method of Undetermined Coefficients

To solve the non-homogeneous equation $a\,y'' + b\,y' + c\,y = g(t)$,

1. Find the solution to the homogeneous equation, y_h.
2. Find a particular solution to the non-homogeneous, y_p, using $g(t)$ to find undetermined coefficients (see below).
3. The general solution to the non-homogeneous equation is given by $y = y_h + y_p$.

The particular solution to the non-homogeneous equation depends on the form of $g(t)$:

	If $g(t)$ has the form	**the particular solution, y_p, will have the form**
1:	$P_n(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0$	$t^s\left(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0\right)$
2:	$P_n(t)\,e^{\alpha t}$	$t^s\left(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0\right)e^{\alpha t}$
3:	$\cos(\beta t)$ and/or $\sin(\beta t)$	$t^s(B_1\cos(\beta t) + B_2\sin(\beta t))$
4:	$P_n(t)\begin{pmatrix}\cos(\beta t)\\ \text{or}\\ \sin(\beta t)\end{pmatrix}e^{\alpha t}$	$t^s\left[\left(A_n t^n + A_{n-1} t^{n-1} + \cdots + A_0\right)e^{\alpha t}\cos(\beta t) + \left(B_n t^n + B_{n-1} t^{n-1} + \cdots + B_0\right)e^{\alpha t}\sin(\beta t)\right]$

(Choose s, so that there are no common terms in y_p and y_h. s will be 0, 1, or 2}

Example 2 Find the general solution to the equation $y'' - 5y' + 6y = 2t^2 + 3$

Example 3 Find the general solution to the equation $y'' + 3y' - 4y = 10\sin(2t)$

Example 4 Find the general solution to the equation $y'' - 6y' + 9y = 8e^{3t}$

Example 5 Find the solution to the equation $y'' + 4y = 4\cos(2t)$, $y(0) = 10$ and $y'(0) = 1$