3.4 Repeated Roots; Method of Reduction of Order

Example 1 Use the characteristic equation to find two linear independent solutions to the equation $y'' - 4y' + 4y = 0$.

Example 2 Suppose there is a second solution to the equation in *Example 1* in the from $y_2 = v\,e^{rt}$ for some function $v(t)$. Find v and the simplest form for the family of solutions to the equation. (Are y_1 and y_2 linearly independent?)

Example 3 Solve the IVP $16y'' + 8y' + y = 0$, $y(0) = 10$, $y'(0) = 4$, and graph. Find a y_0' that separates the solutions that stay positive from those that tend to negative.

Manipulate

Reduction of Order

The method used in *Example 2* is called *Reduction of Order*. If one solution to a differential equation is known, i.e. $y_1(t)$, a second solution may be found by letting $y_2(t) = v(t)\, y_1(t)$. Using $L[y_2(t)] = 0$ we can solve for $v(t)$ (which is a first order linear equation in terms of the function $v'(t)$).

"Proof"

Example 4 Suppose one solution to the differential equation $t^2 y'' + t y' - 9 y = 0$ is $y_1(t) = t^3$, use reduction of order to find a second linearly independent solution.