3.1 Second Order Equations

Second order ordinary differential equations have the form $\frac{d^2y}{dt^2} = f\left(t,\ y,\ \frac{dy}{dt}\right)$. It is linear if the right hand side can be written $f\left(t,\ y,\ \frac{dy}{dt}\right) = g(t) - p(t)\,y - q(t)\,y'$, or

$$y'' + q(t)\,y' + p(t)\,y = g(t)$$

We also often see this as

$$P(t)\,y'' + Q(t)\,y' + R(t)\,y = G(t).$$

If P, Q, and R are constants and $G = 0$, then the equation is a *second order linear homogeneous equation with constant coefficients*.

Second Order Linear Homogeneous Equations with Constant Coefficients

Example 1 Find two solutions to the differential equation $y'' - y = 0$. (Is there a function whose second derivative equals itself?)

Example 2 For the equation in example 1, find the particular solution if $y(0) = 4$ and $y'(0) = -1$

Example 3 Show that $y = e^{rt}$ is a solution to $a\,y'' + b\,y' + c\,y = 0$, for suitable values a, b, c, and r.

The Characteristic Equation

For the differential equation with real constant coefficients $a\,y'' + b\,y' + c\,y = 0$, the **characteristic equation** is

$$a\,r^2 + b\,r + c = 0$$

If the zeros of the polynomial, r_1 and r_2, are real and not equal, the general solution is

$$y = C_1\,e^{r_1 t} + C_2\,e^{r_2 t}.$$

Example 5 Find two solutions, y_1 and y_2, to the equation $y'' - y' - 6\,y = 0$, and show that $y = C_1\,y_1(t) + C_2\,y_2(t)$ is also a solution.

Example 6 Solve the IVP for $8\,y'' + 6\,y' + y = 0$; $y(0) = 5$ and $y'(0) = 2$. Graph the solution; find the long term behavior and the maximum value of the solution.