2.3 Modeling with First Order Linear Equations

Example 1 The rate of change in a population is proportional to the population. Write the differential equation for this relation and solve the equation.

Example 2 The logistic equation from Section 1.1 was $\frac{dP}{dt} = bP\left(1 - \frac{P}{K}\right)$. Solve for P. *(Also, find when the rate of change in the population is the greatest.)*

Example 3 A 500 L tank contains 1 kg of copper-chlorate. An inlet pipe is filling the tank with a solution with concentration 10 g/L copper-chlorate at a rate of 20 L per minute. An outlet pipe is draining the well-mixed solution also at a rate of 20 L/min. Find the amount (and concentration) of copper-chlorate after 10 minutes, and find the long-term amount of copper-chlorate in the tank.

Example 4 A 2000 liter tank currently has a concentration of 20 grams per liter of ammonia and an initial volume of 1200 liters. Liquid is being drained at a rate of 5 liters per minute while a solution of 5 grams per liter of ammonia is entering at a rate of 10 liters per minute. Find the quantity of ammonia in the tank when it becomes full of solution. (Also find the concentration to check your answer.)

Example 5 Two tanks, A and B, are coupled, in that, an inlet pipe flows into tank A, tank A then flows into tank B, and an outlet pipe flows out of B. Tank A has volume 500 gallons, and Tank B has volume 800 gallons. The initial concentration of an acid in tank A is 8 ounces per gallon, and the initial concentration of acid in tank B is 2 ounces per gallon. Fresh water is being pumped into tank A at a rate of 20 gallons per minute, and drained from tank B also at 20 gallons per minute. (Assume A flows into B at the same rate.)

a) Find the concentration in each tank after 10 minutes.
b) At what time will the concentration be equal?
c) What is the maximum concentration in tank B?
d) At what time will tank B have a concentration less than 1 ounce per gallon?