# 2.2 Separable Equations

The standard form for a first order differential equation is $y' = f(x, y)$. Many (but not all) first order equations can be written in this form by isolating $y'$, and then setting the right-hand side equal to $f(x, y)$.

**Example 1** Write the equation $y'x + 4y\,y' + x = xy$ in standard form.

Note: in the equation above, if $-M(x, y)$ is the numerator, and $N(x, y)$ is the denominator we have $y' = \frac{-M(x,y)}{N(x,y)}$ which can be written

$$M(x, y) + N(x, y)\frac{dy}{dx} = 0$$

or in *differential form*

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

## Separable Equations

If $M$ is strictly a function of $x$ and $N$ is strictly a function of $y$, then the equation is said to be **separable** and can be written

$$M(x)\,dx + N(y)\,dy = 0$$

**Example 2** Solve the initial value problem $y' = x\,y^3\left(1 + x^2\right)^{-1/2}$ and $y(0) = 1$. Determine the interval in which the solution exists.

**Example 3** Solve the initial value problem $y' = 2(1+x)\left(1+y^2\right)$, $y(0) = 0$, and find the minimum value of the solution.

**Example 4** Solve the differential equation $y' = \frac{x^2+3y^2}{2xy}$. Note: the equation is separable using the substitution $y = vx$. Solve the IVP $y(1) = 1$ and graph the integral curve. Does it match the direction field?