2.1 First Order Linear Differential Equations - Integrating Factors

A first order differential equation can be written as $y' = f(t, y)$. A solution to the differential equation is any function $y = \phi(t)$ that satisfies the equation for all t in an interval. Usually, an arbitrary equation $y' = f(t, y)$ has no clear method to find the solution. This chapter looks at various types of first order equations that do have a clear method to solve.

First Order Linear Equations; Method of Integrating Factors.

A first order linear equation can always be written in the form

$$y' + p(t)\,y = q(t)$$

While this equation doesn't always have a direct method to find the solution, multiplying the entire equation by a function $\mu(t)$ (called an integrating factor), does allow us to directly solve the equation (provided μ exists). The problem is finding the integrating factor μ. So, suppose we solve the general first order linear equation by multiplying by a function μ:

$$y' + p(t)\,y = q(t)$$

Integrating Factor

The first order linear differential equation $y' + p(t)\,y = q(t)$ has integrating factor:

and the solution is found by solving the equation: for y.

Example 1 Find the general solution to the first order linear differential equation: $y' + 2y = te^{-t}$

Example 2 Find the integrating factor and the solution to the differential equation: $ty' + 3y = t + 2$ given $y(1) = 2$.

Example 3 Solve the IVP: $\frac{dQ}{dt} = 20 - \frac{Q}{25+2t}$ where $Q(0) = 1000$. (When is Q a minimum?)