1.3 Classification of Differential Equations

Ordinary Linear Differential Equations

Functions of a single variable, e.g., $y = f(x)$

$$y''(x) - x\,y'(x) + y(x) = 5\cos(x)$$

These can always be written as $F\big(x,\ y'(x),\ y''(x),\ \ldots y^{(n)}(x)\big) = 0$. Typically we will only be dealing with functions that can be written $y^{(n)}(x) = G\big(x,\ y'(x) \ldots y^{(n-1)}(x)\big)$.

Partial Differential Equations

These involve functions of several variables, i.e., $u(x,\ y) = x^2\,y + x\,e^y$. Here, we can take the derivative with respect to x and with respect to y. An example is the *Laplace* equation:

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0$$

Ordinary Non-Linear Equations

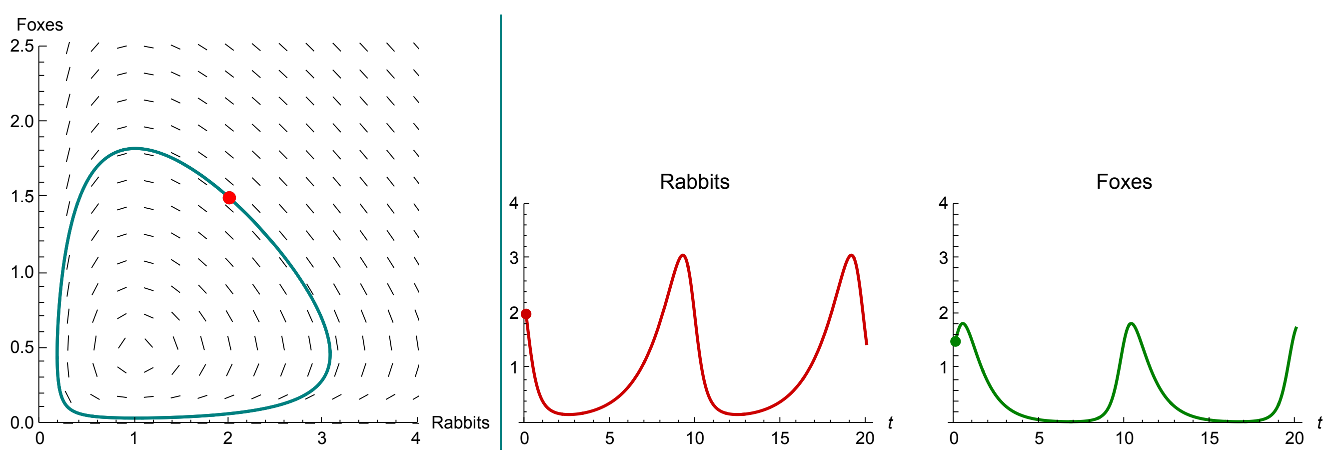
A linear differential equation only involves **terms** of the unknown function y and its derivatives. Any DE including terms such as $(y')^2$, $y\,y'$, $\ln(y)$, etc., are non-linear terms, and hence would make non-linear differential equations.

Systems of Differential Equations

One of the most common examples of a system of differential equation is the *Lotka-Volterra* equations. These describe the dynamics in the populations of a prey $x(t)$ (e.g., rabbits), and a predator $y(t)$ (e.g. foxes), or more precisely, the rate of change in the populations:

$$\begin{cases} dx/dt = \alpha\,x - \beta\,x\,y \\ dy/dt = -\gamma\,y + \delta\,x\,y \end{cases}$$

The parameters α, β, γ, δ are usually found empirically. For example, if we let $\alpha = 0.6$, $\beta = 1.3$, $\gamma = \delta = 1$, and the initial population is $x = 2$ and $y = 1.5$ (in thousands), we get the following direction field and integral curve.



Copyright © 2016 CStevens

Example 1 Show that the differential equation $t^2 y'' + 5ty' + 4y = 0$ has solutions $y_1 = t^{-2}$ and $y_2 = t^{-2}\ln(t)$.

Example 2 Find the value(s) of r such that $y = t^r$ is a solution to the equation $t^2 y'' - 4ty' + 4y = 0$.

Example 3 Find the value of r such that $y = xe^{rx}$ is a solution to $y'' - 6y' + 9y = 0$ for all x.

Copyright © 2016 CStevens