1.2 Solutions to Differential Equations

Solutions to Differential Equations

A *solution* to a differential equation is a function $y = \phi(x)$ that when substituted into the equation along with its derivatives results in an identity.

Example 1 Show that the function $y = 3xe^x$ is a solution to the differential equation $y'' = 2y' - y$ (a *Second Order - Homogeneous - Linear - differential equation - with constant coefficients*).

In fact, it can be shown that **any** function of the form $y = C_1 e^x + C_2 x e^x$ is a solution to the DE, regardless of the constants C_1 and C_2. This is called a **general solution**. The constants are determined by appropriate *initial conditions*, i.e., if $y(0) = 4$ and $y'(0) = 2$, then $y = 4e^x - 2xe^x$ is a **particular solution** for the DE with the given initial conditions.

Example 2 For the object in free fall in example 1.1.5, find the *general* solution to the differential equation. Recall: $v' = \frac{dv}{dt} = 9.8 - 0.25v$.

Example 3 Suppose the object in *Example 2* is dropped from a height of 200 meters. At what time *t* does the object hit the ground, and at what velocity does it hit the ground?

Example 4 *Newton's Law of Cooling* states the rate of change in the temperature of an object is proportional to the difference in its temperature and the surrounding temperature. A piece of metal with an initial temperature of 200°C is placed into a room with ambient temperature 30°C. Find the general solution to the differential equation, and how long it will take the object to reach 40°C if its temperature is 180°C after 10 minutes.

Example 5 Find the general solution to the unbounded population growth problem (*the rate of change of a population is proportional to the population size*).