Math 238 - Differential Equations

1.1 Why Differential Equations?

A *differential equation* is an equation that contains an unknown function, and one or more derivatives of the unknown function. The *solution* to the differential equation is the equation in which the function and its derivatives satisfy the equation when substituted into the equation. The following are examples of differential equations.

Example 1 The rate of change of a population is proportional to the size of the population:

$$\frac{dP}{dt} = kP$$

Example 2 The restorative force to bring a spring back to its original length is proportional to the distance stretched: $F = -kx$. But, since $F = ma$, or equivalently $F = mx''$, we have

$$mx'' = -kx$$

Example 3 *Newton's Law of Cooling* states the rate of change in the temperature of an object is proportional to the difference in its temperature and the surrounding temperature

$$\frac{dT}{dt} = k(T - T_s)$$

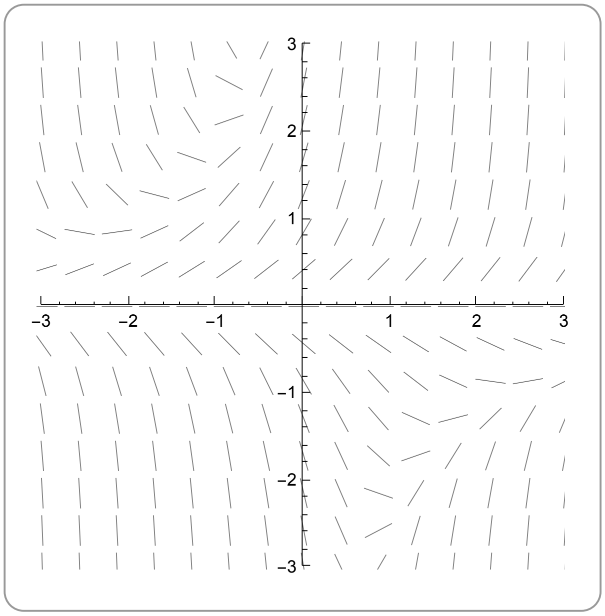
Example 4 A *logistic growth* model assumes that if a population is zero the growth rate of the population is zero, and if a population reaches a maximum value K the growth rate also is zero. One possible function is

$$\frac{dP}{dt} = bP\left(1 - \frac{P}{K}\right)$$

Example 5 Develop an equation modeling the free fall of an object assuming $F = ma$ and the fact that wind resistance is marginally proportional to velocity. The wind resistance is called the drag force and is often denoted γ.

Direction Fields

Consider the differential equation $y' - y^2 x = 2y$. Solving for y' allows us to create a **direction field** (or slope field) by calculating the slope (y') at various points (x, y) in the plane, and plotting short line segments at these points. Solution curves through an initial point can then be estimated. Some sample points and solution curves are given below.



Direction Field for $y' = y^2 x + 2y$

Example 6 Using the differential equation developed in Example 5, An object has a mass of 8 kg, and a drag coefficient $\gamma = 2$. Find a differential equation to create a corresponding slope field. (assume $g = 9.8\ \frac{m}{s^2}$.) (Also, find the units for γ.)

Direction Fields with Mathematica

```
VectorPlot[{1, 9.8 - 0.25 v}, {t, 0, 10}, {v, 0, 80},
 VectorScale → {Scaled[.03]},
 VectorPoints → 15,
 VectorStyle → {"Segment", Gray},
 Frame → False,
 Axes → True,
 PlotRange → {{0, 10}, {0, 80}}
]
```