4.2 Null Space, Column Spaces

Example 1 Solve the system of equations: $\begin{cases} 2x_1 + x_2 - 2x_3 = 0 \\ x_1 - 2x_2 + 9x_3 = 0 \end{cases}$

Definition

The **null space** of an $m \times n$ matrix , written as Nul A, or $N(A)$, is the set of solutions to the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

Example 2 Show that the null space of example (1) is a subspace. (Show closure under addition and scalar multiplication.)

Theorem 2

The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$. Note, the null space of a matrix A is often referred to as the **kernel** of A.

Example 3 Show that the plane P: $x + 3y - 4z = 0$ is a subspace, and show that P is also null space.

Example 4 Find a spanning set for the null space of the matrix $\begin{pmatrix} 1 & 3 & 0 & 2 & 3 \\ 1 & 3 & 1 & 1 & 5 \\ 2 & 6 & 1 & 3 & 8 \end{pmatrix}$. If the null space is in $\mathbb{R}^n$ what is n?

Definition

The **column space** of an $m \times n$ matrix, written Col (A), is the set of all linear combinations of the columns of A.

That is $\text{Col}(A) = \{\mathbf{b} \mid \mathbf{b} = A\,\mathbf{x} \text{ for some } \mathbf{x} \text{ in } \mathbb{R}^n\}$.

Example 5 Find A such that the given set is col(A)

$$\left\{ \begin{pmatrix} s + 4t \\ r + 3s - 3t \\ 3r + t \end{pmatrix} \middle|\, r, s, t \in \mathbb{R} \right\}$$

Example 6 Find the null space and column space for the matrix $A = \begin{pmatrix} 1 & 3 & 1 & 4 \\ 2 & 7 & 3 & 0 \\ 1 & 5 & 3 & 1 \\ 1 & 2 & 0 & 8 \end{pmatrix}$.