3.2 Properties of Determinants

From example 5 in 3.1 notes, we have the following theorem

Theorem 3

Let A be a square matrix.

a. If a multiple of a row of A is added to another row to produce matrix B, then $\det(B) = \det(A)$
b. If two rows of A are interchanged to produce matrix B, then $\det(B) = -\det(A)$.
c. If one row of A is multiplied by k to produce matrix B, then $\det(B) = k\det(A)$.

Example 1 Compute the determinant of A using row reduction: $A = \begin{pmatrix} 3 & -2 & 4 & 1 \\ 6 & 4 & 0 & 14 \\ 0 & 0 & 0 & 2 \\ 6 & -4 & 5 & 5 \end{pmatrix}$

Theorem 4

A square matrix is invertible if and only if $\det(A) \neq 0$.

"Proof":

Theorem 5

If A is an $n \times n$ matrix, then $\det(A) = \det(A^T)$

What does this imply for Theorem 3?

Theorem 6

If A and B are $n \times n$ matrices, then $\det(AB) = \det(A)\det(B)$.

Example 2 Verify theorem 6 for $A = \begin{pmatrix} 3 & 2 \\ 5 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 5 \\ 1 & 3 \end{pmatrix}$.

Example 3 True or False $\det(A + B) = \det(A) + \det(B)$?