

## 2.7 Computer Graphics

In this section we want to explore transformation matrices that pertain to computer graphics, particularly scaling, shearing, rotation, and translation.

### Scaling Transformations

**Example 1** Find the matrix that scales a point from  $(3, 2)$  to  $(12, 2)$ .

**Example 2** Find the matrix that scales a point from  $(-2, 4)$  to  $(-2, -8)$ .

### Shearing Transformations

**Example 3** Find the matrix that shears a point horizontally proportional to 0.5 times the  $y$  coordinate.

**Example 4** Find the matrix that shears a point vertically proportional to 2 times the  $x$  coordinate.

### Rotation Transformation

**Example 5** Find the matrix that rotates a point (e.g., a vector)  $60^\circ$  counter clockwise (or  $\frac{\pi}{3}$  rad). Find the coordinates of  $(5, 6)$  after this rotation.

## Composite Transformations

**Example 6** Find a matrix that scales a point horizontally by 5, vertically by 2, and rotated by  $180^\circ$ .

## Translation Matrices

Is a vertical or horizontal translation linear? Why or why not?

## Homogeneous Coordinates

A point  $(x, y)$  in  $\mathbb{R}^2$  can be written in *homogeneous coordinate* form  $(x, y, 1)$ , or in vector form  $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$  where the 3rd element is ignored when graphing. The transformation matrix also has to be rewritten as a  $3 \times 3$  matrix, e.g., the scaling matrix  $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$  is written as  $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ . Using homogeneous coordinates a point in  $\mathbb{R}^2$  can be horizontally or vertically translated.

**Example 7** Show that the transformation matrix  $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  translates the point  $(3, 5)$  horizontally 3 units. First convert the point to homogeneous coordinates, and then multiply.

**Example 8** Find the transformation matrix that translates a point  $(x, y)$  5 units to the right and 2 units up.

## Using *Mathematica* for Graphics Manipulation