2.1 Matrix Operations

Theorem 1

Let A, B, and C be matrices of equal size, and let r and s be scalars. Then

a. $A + B = B + A$
b. $(A + B) + C = A + (B + C)$
c. $A + 0 = A$
d. $r(A + B) = rA + rB$
e. $(r + s)A = rA + sA$
f. $r(sA) = (rs)A$

Matrix Multiplication

The matrix multiplication AB can be written as $A[\mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \cdots\ \mathbf{b}_p] = [A\mathbf{b}_1\ A\mathbf{b}_2\ A\mathbf{b}_3\ \cdots\ A\mathbf{b}_p]$

Example 1 For $A = \begin{pmatrix} 2 & 6 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 0 & 4 \end{pmatrix}$ use the above definition to find AB.

Row-Column Method of Matrix Multiplication

When multiplying two matrices the element in the i^{th} row and j^{th}column, e_{ij} is found by multiplying across the i^{th} row of A and the j^{th} column of B.

Example 2 Find the product AB by hand: $A = \begin{pmatrix} 2 & 3 & -1 \\ -3 & 6 & 2 \\ 7 & 0 & -5 \\ 3 & 2 & 9 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ -2 & 6 \\ 3 & 8 \end{pmatrix}$.

Theorem 2: Properties of Matrix Multiplication

Let A be an $m \times n$ matrix, and let B and C have dimensions in which the following sums and products are defined.

a. $A(BC) = (AB)C$
b. $A(B + C) = AB + AC$
c. $(B + C)A = BA + CA$
d. $r(AB) = (rA)B = A(rB)$ for any scalar r.
e. $I_m A = A = A I_n$

Example 3 Let $A = \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix}$. Find AB and BA.

Example 4 Let $A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$. Find A^3.

Definition

The **transpose** of a matrix, A denoted A^T is a matrix whose columns are the rows of A.

Example 5 Let $A = \begin{pmatrix} 4 & -3 & 2 \\ 1 & 5 & 4 \end{pmatrix}$, find A^T.

Theorem 3

Let A and B have appropriate dimensions for the following sums and products.

a. $(A^T)^T = A$
b. $(A + B)^T = A^T + B^T$
c. $(rA)^T = rA^T$ for any scalar r.
d. $(AB)^T = B^T A^T$

Example 6 Let $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -4 & 3 \\ 6 & 2 & -1 \end{pmatrix}$. Which of the following are possible? (Simplify expressions e, f, and g.)

a. AB b. BA c. $A^T B$ d. $B^T A$ e. $B^T(AB^T)^T$ f. $B(AB^T)^T$ g. $A(A + (BB^T))^T$