1.5 Solution Sets of Linear Systems

Definition

A **homogeneous** system of linear equations is any equation that can be written in the form: $A\mathbf{x} = \mathbf{0}$.

Example 1 Determine if the homogeneous system has a non-trivial solution, i.e., $\mathbf{x} \neq \mathbf{0}$

$$\begin{aligned} -1x_1 + 3x_2 + x_3 &= 0 \\ 2x_1 - x_2 + 3x_3 &= 0 \\ 3x_1 - 4x_2 + 2x_3 &= 0 \end{aligned}$$

What must be true for a homogeneous system to have a non-trivial solution?

Example 2 For the system in example 1, write the solution in **vector form** and describe the solution geometrically.

Example 3 Write the solution to the equation $4x_1 - 7x_2 - 6x_3 = 0$ in vector form and describe the solution in terms of "Span".

Solutions of Nonhomogeneous Systems

Example 4 Solve the homogeneous equation $A\,\mathbf{x} = \mathbf{0}$ for $A = \begin{pmatrix} 1 & 0 & -3 \\ 2 & 2 & 4 \\ 1 & 2 & 7 \end{pmatrix}$, and write the solution in parametric vector form.

Example 5 Describe the solution set for for the non-homogeneous system of equations: $\begin{cases} x_1 \qquad\quad - 3\,x_3 = 2 \\ 2\,x_1 + 2\,x_2 + 4\,x_3 = 6 \\ x_1 + 2\,x_2 + 7\,x_3 = 4 \end{cases}$.

Write the solution in the form $\mathbf{x} = \mathbf{p} + t\,\mathbf{v}_h$.

Theorem 6

Suppose the equation $A\,\mathbf{x} = \mathbf{b}$ is consistent for some given $\mathbf{b}$, and let $\mathbf{p}$ be a solution. Then, the solution set of $A\,\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is any solution to the homogeneous equation $A\,\mathbf{x} = \mathbf{0}$.