13.2 Derivatives and Integrals of Vector Functions

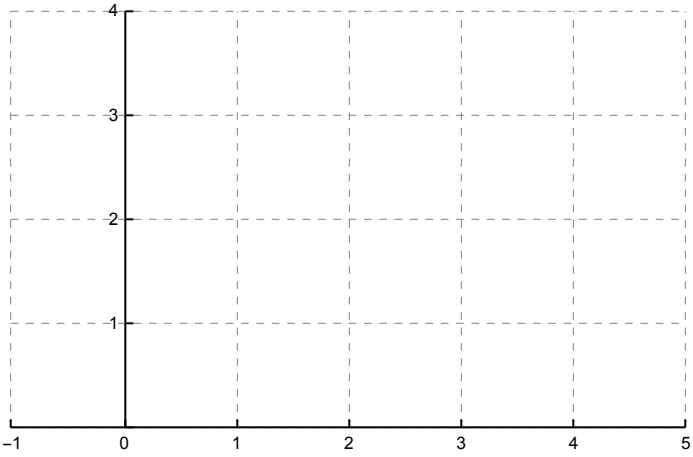
Differentiation

The derivative of a vector function is: $\boldsymbol{r}'(t) = f'(t)\,\boldsymbol{i} + g'(t)\,\boldsymbol{j} + h'(t)\,\boldsymbol{k}$.

Example 1 Find $\boldsymbol{r}'(t)$ for $\boldsymbol{r}(t) = \sin^{-1}(t)\,\boldsymbol{i} + e^{-2t}\,\boldsymbol{j} + \sqrt{1-t}\;\boldsymbol{k}$.

Example 2 (a) Graph the function $\boldsymbol{r}(t) = t^2\,\boldsymbol{i} + (t+1)\,\boldsymbol{j}$.

(b) Find $\boldsymbol{r}'(t)$ and plot $\boldsymbol{r}(1)$ and $\boldsymbol{r}'(1)$.



Example 3 Find the unit tangent vector $\boldsymbol{T}(t) = \dfrac{\boldsymbol{r}'(t)}{\|\boldsymbol{r}'(t)\|}$ for $\boldsymbol{r}(t) = e^{2t}\,\boldsymbol{i} + \sin(t)\,\boldsymbol{j} + \left(t^2+1\right)\boldsymbol{k}$ for $t = 0$.

Example 4 Find the tangent line to the function $\boldsymbol{r}(t) = t\,\boldsymbol{i} + t^2\,\boldsymbol{j} + t^3\,\boldsymbol{k}$ when $t = 2$.

Example 5 A function is smooth if $\boldsymbol{r}'(t) \neq \boldsymbol{0}$ for all t. Where is the function $\boldsymbol{r}(t) = \left(2t - t^2\right)\boldsymbol{i} + \left(\frac{t^3}{3} - t\right)\boldsymbol{j}$ not smooth?

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Example 6 Let $\boldsymbol{u}(t) = t^2\,\mathbf{i} + 3t\,\mathbf{j} - \mathbf{k}$ and $\mathbf{v}(t) = t\,i - 2t^3\,\mathbf{j} + (2t-1)^2\,\mathbf{k}$. Find $\frac{d}{dt}(\mathbf{u}(t) \times \mathbf{v}(t))$ when $t = 1$

Integration

The definite integral of a vector function is: $\int_a^b \boldsymbol{r}(t)\,dt = \left(\int_a^b f(t)\,dt\right)\boldsymbol{i} + \left(\int_a^b g(t)\,dt\right)\boldsymbol{j} + \left(\int_a^b h(t)\,dt\right)\boldsymbol{k}$

Example 7 Evaluate $\int_1^4 \left(\sqrt{t}\;\boldsymbol{i} + e^{-t}\,\boldsymbol{j} + \frac{1}{t^2}\,\boldsymbol{k}\right)dt$