13.1 Vector-Valued Functions

A vector-valued function is a function of t resulting in a vector $\boldsymbol{r}$ where each component in $\boldsymbol{r}$ is a function of t. That is,

$$\boldsymbol{r}(t) = \langle f(t),\ g(t),\ h(t)\rangle = f(t)\,\boldsymbol{i} + g(t)\,\boldsymbol{j} + h(t)\,\boldsymbol{k}$$

The Limit of a Vector-Valued Function

If $\boldsymbol{r}(t) = \langle f(t),\ g(t),\ h(t)\rangle$, then

$$\lim_{t\to a} \boldsymbol{r}(t) = \left\langle \lim_{t\to a} f(t),\ \lim_{t\to a} g(t),\ \lim_{t\to a} h(t) \right\rangle$$

provided the limits of the component functions exist.

Example 1 Find the domain of the function $\boldsymbol{r}(t)$, and $\lim_{t\to 0} \boldsymbol{r}(t)$ for the function $\boldsymbol{r}(t) = \left\langle \frac{3}{t-3},\ \ln(t+1),\ \frac{\sin(t)}{t} \right\rangle$.

The graph of the vector-valued function $\boldsymbol{r}(t) = f(t)\,\boldsymbol{i} + g(t)\,\boldsymbol{j} + h(t)\,\boldsymbol{k}$ is a space curve with parametric equations $\{x = f(t),\ y = g(t),\ z = h(t)\}$.

Example 2 Sketch the function (space curve) $\boldsymbol{r}(t) = 2\cos(t)\,\boldsymbol{i} + 2\sin(t)\,\boldsymbol{j} + t\,\boldsymbol{k}$.

Example 3 Find the equation of the curve of intersection of the surface $x = y^2$ and the plane $2x + 4y + 3z = 3$.

Example 4 Describe the curve given by $\boldsymbol{r}(t) = 4\cos(t)\,\boldsymbol{i} + 4\sin(t)\,\boldsymbol{j} + 3\cos(7t)\,\boldsymbol{k}$.

Example 5 Sketch the space curve $r(t) = \sqrt{t}\,\boldsymbol{i} + t^2\,\boldsymbol{j} + t\,\boldsymbol{k}$.