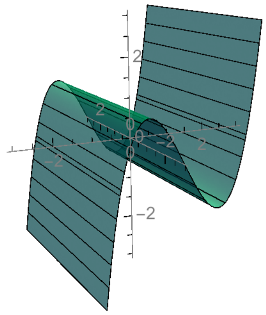
12.6 Cylinders and Quadric Surfaces

A **cylinder** is a surface that consists of all lines (called rulings) that are parallel to a given line and pass through a given plane curve. An example is the graph if $z = x^3 - 2x$.



Notice that the variable y is missing from the equation of the surface. This is typical of a cylinder; any one if the variables, x, y, or z can be missing.

Example 1 Graph and describe the cylinder $x = y^2 - 3$

Quadric Surfaces

A quadric surface is a surface that is at most quadratic in x, y, and z. That is, in general form:

$$A x^2 + B y^2 + C z^2 + D x y + E x z + F y z + G x + H y + I z + J = 0$$

These surfaces are analogous to the conic sections in two dimensions. Using translations and rotations these can be rewritten into one of two standard forms:

$$A x^2 + B y^2 + C z^2 + J = 0 \quad \text{or} \quad A x^2 + B y^2 + I z = 0$$

The best way to visualize or graph the surfaces is to look at the traces, i.e., $x = k$, $y = k$, and/or $z = k$, for various values of k.

Example 2 Describe the traces and sketch the quadric surface $x^2 + 3z^2 = y$; *an elliptic paraboloid*.

Example 3 Use traces to sketch the quadric surface $2x^2 + 2y^2 - z^2 = 8$; a *hyperboloid of one sheet*.

Example 4 Sketch the quadric surface $-\frac{x^2}{4} - \frac{y^2}{9} + z^2 = 1$; a *hyperboloid of two sheets*.

Example 5 Sketch the quadric surface $\frac{x^2}{2} - \frac{y^2}{4} = z$; a *saddle*.