12.1 Three-Dimensional Coordinate System

Example 1 Draw the $x\,y\,z$ – coordinate system, and plot the points $(3, 6, 4)$ and $(4, -2, 7)$.

Example 2 Make a sketch of the surfaces $z = 5$, and $y = 2$.

The Distance Formula in $\mathbb{R}^3$

Given two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ the distance between P_1 and P_2 is given by

$$| P_1 P_2 | =$$

Example 3 Find the distance between the points plotted in Example 1.

Example 4 Find the distance between the point (h, k, l) and the general point (x, y, z).

Equation of a Sphere

The equation of a sphere with center (h, k, l) and radius r is:

Example 5 Find the center and radius of the sphere: $x^2 + y^2 + z^2 - 10x + 4y - 14z + 69 = 0$

Example 6 The sphere in Example 5 intersects one of the coordinate planes (xy-plane, xz-plane or yz-plane). Find the trace (or equation) of the intersection of the sphere and the plane.

Example 7 Describe the region given by $4 \le x^2 + y^2 + z^2 \le 25$ AND $x \le 0$.