11.8 Power Series

A **Power Series** is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots$$

Example 1 For what values of x does the series $\sum_{n=0}^{\infty} x^n$ converge?

A more general power series centered at a is given by

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \ldots$$

Example 2 Find the values for which the power series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n+1}$ converges.

Example 3 For what values does the series $\sum_{n=0}^{\infty} n!\,(x-2)^n$ converge?

Example 4 Find the values for which the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converge.

Theorem

For a given power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only three possibilities:

1. The series converges only at $x = a$.
2. The series converges for all x.
3. The series converges on a set of values such that $|x-a| < R$ and diverges when $|x-a| > R$ The value R is called the **radius of convergence** with the center at a, and the interval of convergence are the values $a - R < x < a + R$. **Note**: the endpoints of the interval must be checked separately.

Example 5 Find the interval of convergence for $\sum_{n=0}^{\infty} 2\left(\frac{x}{5}-4\right)^n$.

Example 6 Find the radius and interval of convergence for $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2 3^n}$

Example 7 Find the domain of the Bessel function of order 1 given by $J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!\,(n+1)!\,2^{2n+1}}$. Find expressions for the first several partial sums.