11.6 The Ratio and Root Test

The Ratio Test

Let $\sum a_k$ be an infinite series with positive terms and let $r = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right|$.

1. If $r < 1$, the series converges.
2. If $r > 1$, the series diverges.
3. If $r = 1$, the test is inconclusive.

Example 1 Determine the convergence $\sum_{k=1}^{\infty} \frac{k\,2^k}{k!}$

Example 2 Determine whether each series converges or diverges.

a. $\sum_{n=0}^{\infty} \frac{n^2\,2^{n+1}}{3^n}$

b. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

Example 3 Determine the convergence of $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1}$.

The Root Test

Let $\sum a_k$ be an infinite series and let $\rho = \lim_{k\to\infty} \sqrt[k]{|a_k|}$

1. If $\rho < 1$, the series converges absolutely.
2. If $\rho > 1$, the series diverges.
3. If $\rho = 1$, the test is inconclusive.

Example 4 Determine the convergence or divergence of $\sum_{k=1}^{\infty} \frac{e^{2k}}{k^k}$.

Example 5 Determine the convergence of $\sum_{k=2}^{\infty} \left(\frac{k^3+2k}{3k^3-1}\right)^k$