11.5 Alternating Series

Recall the harmonic series: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots = \sum_{k=1}^{\infty} \frac{1}{k}$ diverges.

Now consider the **alternating harmonic series**: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}$

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}$$

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Log[2]
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Alternating Series Test

The alternating series $\sum(-1)^k a_k$ converges provided

1. the terms of the series are non-increasing in magnitude, or $a_k \geq a_{k+1}$ for all k

2. $\lim_{k\to\infty} a_k = 0$

Example 1 Determine the convergence of $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{k^2}{2^k}$.

Example 2 Does the series in example 1 converge if it is not alternating?

Absolute Convergence

Consider an convergent alternating sequence $\sum(-1)^k a_k$. If the corresponding non-alternating sequence $\sum a_k$ also converges we say the series is **absolutely convergent**. When the corresponding non-alternating sequence diverges we say the original sequence converges conditionally.

Example 3 Determine if $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln(n)}{n}$ converges absolutely.

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Example 4 Determine if the series converges absolutely: $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{\sqrt{k+1}}{k+2}$

Error or Remainder for a Partial Sum of an Alternating Series

Let $S = \sum_{k=1}^{\infty} (-1)^{k-1} a_k$ and $S_n = \sum_{k=1}^{n} (-1)^{k-1} a_k$. The remainder (or error) $R_n = |S - S_n|$ in estimating S with S_n is such that $R_n \le a_{n+1}$. That is, the error is less than the magnitude of the next term in the sequence.

Example 5 Find the error in estimating $S = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{1}{k^3}$ with the first 5 terms. How many terms are needed to estimate S to within 0.001?

Recall for a non-alternating convergent series the error or remainder R_n in estimating S with S_n is $R_n \le \int_n^{\infty} a_x \, dx$. How many terms would be needed to estimate $\sum_{k=1}^{\infty} \frac{1}{k^3}$ to within 0.001.

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