11.1 Sequences and Series

A **sequence** is a list of numbers, i.e, {1, 3, 5, 7, 9,…}, or $\left\{1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \frac{1}{16}, ...\right\}$

Example 1 Find the first four terms of the infinite sequence: $\left\{\frac{n+1}{n!}\right\}_{n=1}^{\infty}$

Example 2 Find the nth term of the sequence: {−7, −3, 1, 5, …}

Example 3 Find the nth term in the sequence: $\left\{2, -2, \frac{4}{3}, \frac{-2}{3}, \frac{4}{15}, \frac{-4}{45}, \frac{8}{315}, \ldots\right\}$

Recurrence relation: $a_{n+1} = f(a_n)$ (Not an explicit formula for each a_n)

Example 4 Find the first 5 terms of the sequence defined by: $a_{n+1} = 2\,a_n - 3$ if $a_1 = 2$, and find an explicit form for a_n.

Convergence of a Sequence

A sequence $\{a_n\}$ has the **limit** L, written

$$\lim_{n\to\infty} a_n = L \quad \text{or} \quad a_n \to L \quad \text{as } n \to \infty$$

if we can make the terms a_n as close to L by taking n sufficiently large. If the limit exists we say the sequence **converges**, otherwise the sequence **diverges**.

Example 5 For the sequence $\left\{\frac{1}{2^n}\right\}$ find N such that $a_n < 0.0001$ for all $n > N$.

Example 6 Find the limit of the sequence $a_n = \frac{2n}{\sqrt{3n^2+1}}$

Example 7 Determine the limit of the sequence $\left\{\frac{(-1)^n n}{n+1}\right\}$.

Monotonic Sequences

Sequences are monotonic if $a_n < a_{n+1}$ for all $n \geq 1$, or $b_n > b_{n+1}$ for all $n \geq 1$.

Example 8 Show that the sequence $a_n = \frac{n+1}{n^2}$ is decreasing

Bounded Sequences

A sequence is **bounded above** if there exists an M such that $a_n \leq M$ for all $n \geq 1$, or **bounded below** if there exists an m such that $m \leq a_n$ for all $n \geq 1$.

Every bounded monotonic sequence is convergent. Why?