

10.4 Areas and Lengths in Polar Coordinates

Areas of Polar Functions

The area of a circle

$$A = \pi r^2$$

The area of a sector of angle θ

$$A = \frac{\theta}{2\pi} \pi r^2$$

$$A = \frac{\theta}{2} r^2$$

If $r = f(\theta)$ and $\theta = \Delta\theta$, then

$$A_i = \frac{\Delta\theta}{2} f(\theta_i)^2$$

and

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} f(\theta)^2 d\theta$$

Example 1 Find the area enclosed by the petals of the rose $r = 4 \cos(3\theta)$

Example 2 Find the area outside the cardioid $r = 1 + \cos(\theta)$ and inside the circle $r = \sqrt{3} \sin(\theta)$.

Example 3 Find the area outside the cardioid $r = 5 + 5 \cos(\theta)$ and inside the limaçon $r = 5 + 3 \sin(\theta)$.

Arc Length of Polar Curves

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

Example 4 Find the length of the limaçon $r = 5 + 3 \sin(\theta)$.