9.1, 9.3 Introduction to Differential Equations, Separable Equations

A *differential equation* is an equation that contains an unknown function, and one or more derivatives of the unknown function. The *solution* to the differential equation is a function in which when the function and its derivatives satisfy the differential equation when substituted in. The following are examples of differential equations.

Example 1 The rate of change of a population is proportional to the size of the population:

$$\frac{dP}{dt} = kP$$

Example 2 The restorative force to bring a spring back to its original length is proportional to the distance stretched: $F = -kx$. But, since $F = ma$, or equivalently $F = mx''$, we have

$$mx'' = -kx$$

Example 3 *Newton's Law of Cooling* states the rate of change in the temperature of an object is proportional to the difference in its temperature and the surrounding temperature

$$\frac{dT}{dt} = k(T - T_s)$$

Example 4 A logistic growth model assumes that if a population is 0 the growth rate is 0, and if a population reaches a maximum value K the growth rate also is zero. One possible function is

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$$

Solutions to Differential Equations

Example 5 Show that the function $y = 3x e^{2x}$ is a solution to the *Second Order - Homogeneous - Linear - with Constant Coefficients* differential equation $y'' - 4y' + 4y = 0$.

We can also show $y = e^{2x}$ is a second solution to the differential equation. In fact, any function of the form $y = C_1 e^{2x} + C_2 x e^{2x}$ is a solution to the differential equation regardless of the values C_1 and C_2.

Example 6 For what value(s) r is $y = e^{r x}$ a solution to the differential equation $2 y'' + 3 y' - 5 y = 0$?

Separable Differential Equations

Example 7 Solve the differential equation: $\frac{dP}{dt} = k P$.

Example 8 Solve the initial value problem: $\frac{y'}{x} = \frac{3 y}{x^2+4}$ given $y(1) = 0$.

Example 9 Solve the initial value problem: $x y' + y = y^2$ given $y(1) = -1$.