8.3 Applications ~ Force and Hydrostatic Pressure

A diver feels a greater pressure the greater the cepth due to the *force* or weight of the column of water above him. The volume of water can be written as Volume = area × depth, or $V = A\,d$. The mass of the water is then the volume multiplied by the density ρ, or $m = A\,d\,\rho$. And finally the force is $F = g\,\rho\,A\,d$ (recall $F = m\,a$), where g is the acceleration due to gravity.

Example 1 Find the total force on a rectangular plate that is 30cm by 50cm when it is horizontal and 3 meters under water.

Example 2 Pressure is defined as force per unit area, or $P = \frac{F}{A} = g\,\rho\,d$, which can be written as $P = \delta\,d$. Find the pressure at any point on the plate in example 1. Note: $1\,N/m^2 = 1\,Pa$ (Pascal).

Force Due to Hydrostatic Pressure

Since $P = \frac{F}{A}$, the force due to hydrostatic pressure is $F = P\,A$. To find the total force due to hydrostatic pressure on the face of a dam, or on a submerged object, we can assume a small slice of area ($length \times \Delta x$) is at the same depth, and hence the same hydrostatic pressure. To calculate the total force, we slice, approximate, and integrate, to get:

$$F = \delta \int \text{length}(x)\,\text{depth}(x)\,dx$$

Example 3 Find the total force on a dam in the shape of a trapezoid that is 20 meters wide at the top, 15-meters at the bottom, and 10-meters deep. $\left(8.17 \times 10^6\,N\right)$

Example 4 A circular window in a submersible has a diameter of 2 feet. Calculate the total force on the window when the submersible is 250 feet below the surface of the water.

Example 5 A gate in the shape of a rectangle with a width of 30 inches and a height of 18 inches is placed so the top is 2 feet below the surface of the water. Find the location $\hat{y}$ on the gate so that the total force above $\hat{y}$ is equal to the total force below $\hat{y}$.

Example 6 A dam is in the shape of an equilateral triangle with each side 4 meters. Find the total force on the face of othe dam if the dam is at an angle of 60° to the horizontal.