# 8.3 Applications ~ Centroids

## Law of the Lever:

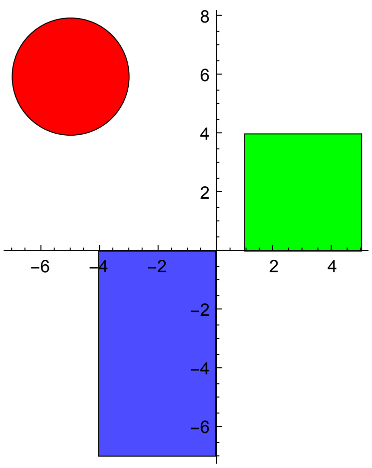
$$d_1\, m_1 = d_2\, m_2$$

**Example 1** Find the pivot point for a 10 foot-lever with 120-lbs on one end and 40-lbs on the other.

Moments : $M_y = \sum m_i\, x_i$ $M_x = \sum m_i\, y_i$ Total Mass : $M = \sum m_i$

$$\overline{x} = \frac{M_y}{M} \quad \overline{y} = \frac{M_x}{M}$$

**Example 2** Find the center of mass of the given system. Assume the shapes are made out of the same uniform density lamina.




**Example 3** The density of a wire is $\rho = \frac{x^2}{50}$ g/cm. Find the center of mass of a 20 cm long piece.

## Centroid of a lamina bounded by $f(x)$

$$M = \rho \int f(x)\, dx \qquad M_y = \rho \int x\, f(x)\, dx \qquad M_x = \frac{1}{2} \rho \int \left(f(x)^2\right) dx$$
$$\overline{x} = \frac{M_y}{M} \qquad \overline{y} = \frac{M_x}{M}$$

**Example 4** Find the centroid of the lamina bounded by $y = x^2 - 2x + 3$ in the first quadrant on the interval [0, 4].

**Example 5** Find the centroid of the lamina bounded by $y = x^2$ and $y = x + 2$.

### Theorem of Pappus

The volume of revolution of a plane region $\mathcal{R}$ is the product of the area of $\mathcal{R}$ and the distance traveled by the centroid of $\mathcal{R}$.

**Example 6** Find the centroid of the lamina bounded by $y = \frac{1}{x}$, $y = 0$, $x = 1$, and $x = 4$.

(a) Find the volume of revolution about $x = -2$.

(b) Find the volume of revolution about the line $y = 2x + 1$. (Pappus' Centroid Theorem)