

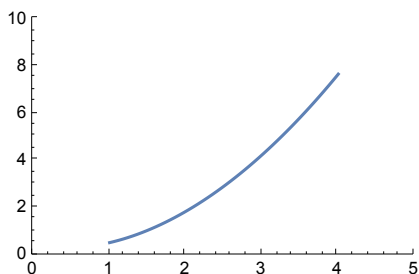
## 8.1 Arc Length

### Arc Length

Let  $y = f(x)$  be a continuous function on the interval  $[a, b]$ . The arc length from  $(a, f(a))$  to  $(b, f(b))$  is given by:

$$L =$$

**Example 1** Find the length of the curve  $f(x) = \frac{x^2}{2} - \frac{\ln(x)}{4}$  from  $x = 1$  to  $x = 4$ .



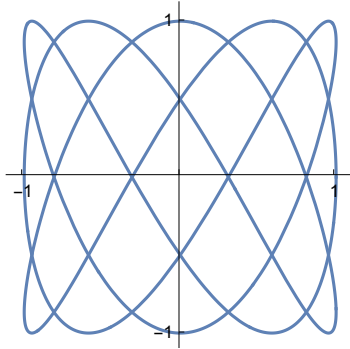
**Example 2** Find the length of the curve  $y = x^{3/2}$  from  $x = 4$  to  $x = 9$ .

## Arc Length of Parametric Curves

Given the parametric function  $F(t) = (x(t), y(t))$ , the length of the curve from  $(x(a), y(a))$  to  $(x(b), y(b))$  is

$$L = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$$

**Example 3** Use your calculator or *Mathematica* to find the length of the Lissajous curve described by:  $x = \cos(3t)$ ,  $y = \sin(5t)$  for  $0 \leq t \leq 2\pi$ .



**Example 4** Find an arc-length function for the curve  $y = \frac{1}{3}x^3 + \frac{1}{4x}$ ,  $x > 0$ , with starting point  $(1, \frac{7}{12})$ .

**Example 5** A baseball is thrown from a height of 5 feet and reaches its highest point of 15 feet when it is 40 feet from the thrower (horizontally).

- Find the distance from the thrower to where the ball hits the ground.
- Find the total distance the ball traveled.