

7.8 Improper Integrals

Type I: Infinite Integrals

$$\int_a^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx = \lim_{t \rightarrow \infty} F(t) - F(a)$$

Example 1 Evaluate the integral: $\int_1^{\infty} \frac{6}{x^3} dx$

Example 2 Evaluate: $\int_{-\infty}^2 x e^x dx$

Example 3 For what values of p does $\int_1^{\infty} \frac{1}{x^p} dx$ converge?

Type 2: Discontinuous Integrands

If f is continuous on either (i) $[a, b)$, or (ii) $(a, b]$, then

$$(i) \int_a^b f(x) dx = \lim_{t \rightarrow b^-} \int_a^t f(x) dx \qquad (ii) \int_a^b f(x) dx = \lim_{t \rightarrow a^+} \int_t^b f(x) dx$$

Example 4 Evaluate: $\int_2^5 \frac{dx}{\sqrt{x-2}}$

Example 5 Evaluate: $\int_0^3 \frac{dx}{x-1}$

Example 6 Torricelli's trumpet (or Gabriel's horn) is created by revolving $y = \frac{1}{x}$ about the x -axis on the interval $[1, \infty)$. Find volume of Torricelli's trumpet.